# Graph-theoretic Performance Analysis for Constellation Link **Geometry via Random Geometric Networks STUDENT: Qishi 'Jackson' Zhou**

# The Big Questions

- How Does ISL range affect satellite network robustness?
- Can simpler network models like random geometric graphs predict the behavior of complex satellite constellations?

### **Our Approach**

- Satellite Constellation Model:
- Based on SpaceX parameters (550 km altitude, 32 planes, 50 satellites per plane
- Constrains: Max degree 4, field of view, reciprocal connections
- Greedy algorithm for link selection
- Random Geometric Graph(RGG) Model:
- Nodes uniformly distributed on a spherical shell
- Max degree 4 is the only constraint
- Same link selection algorithm as satellite constellation
- Key Robustness Metrics Analyzed
- Algebraic Connectivity  $\lambda_2(L(G))$ : Overall network connectivity
- Effectivity Resistance  $R_{eff}(G)$ : Communication efficiency, account for all paths
- Latency (Average Shortest path length  $l_G$ )
- $t_l = \frac{d_{ij}}{c} + l_G t_p$ **Dynamic Weighted Graph Method:**
- Removes all the constrain and link selection algorithm
- - **Key Findings**
- ISL Range & Robustness:
- Increased ISL range improve all robustness metrics
- Saturation points: Beyond a threshold, improvements diminish significantly (sigmoidal trend)
- Satellite Constellation vs. RGGs:
- Remarkable similarity, RGGs show very similar trends in  $\lambda_2(L(G))$ ,  $R_{eff}(G)$  and  $t_l$
- Weighted Graph Analysis:
- Re-confirm similarity in algebraic connectivity between these two networks.
- Both show similar mean and distributional shift with varying weight

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- $W_{ij} = 1 \frac{1}{1 + e^{k(d_{ij} m)}}$



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0.8



#### Weighted Graph Analysis Sigmoid Function: $f(x) = 1 - 1/(1 + e^{k(x-m)})$ 2,000 1.000 3,000 d(km) Satellite Constellation Density Plot --- Mean: 1.856355 Count: 100 Mean: 1.856355 4000 -Median: 1.856566 Std Dev: 0.000488 Min: 1.855401 Min: 2.459444 ---- Random Geometric Graph Max: 1.856863 Max: 2.459813 Satellite Constellation Range: 0.001462 k: -0.003500 -0.003500 ε = 0.05 600 m: 2508.000000 2000 ε = 0.1 ε = 0.2 ε = 0.3 2.459400 1.855000 1.857000 1.857500 1.856000 1.856500 Values Random Geometric Graph Density Plot --- Mean: 1.896 Count: 100 Count: 100 Mean: 1.896 Mean: 2.514 Median: 2.519 Median: 1.901 Std Dev: 0.064 Std Dev: 0.058 Min: 2.356 Min: 1.730 Max: 2.653 Max: 1.992 Range: 0.262 Range: 0.297 k: -0.003500 k: -0.003500 0.8 1.0m: 2508.00000 m: 2708.000000 --- Random Geometric Graph ---- Satellite Constellation ε = 0.05 ε = 0.1 1.65 2.05 1.70 1.75 1.85 1.90 1.95 2.00 1.80 2.3 Values ε = 0.2 ε = 0.3

# **Conclusion & Future Works**

- ISL range is a critical design parameter with a clear saturation point for robustness gain
- RGGs demonstrate similarities to complex stellation constellation
- This suggest RGGs can serve as computationally efficient proxies in early-stage satellite constellation design
- Develop analytical results to formally explain the observed similarity
- Investigate the impact of orbital perturbations and other real world complexity

