Estimation-Aware Trajectory Optimization with Set-Valued Measurement Uncertainties

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Background

- Trajectory optimization under sensor uncertainty may require the user to restructure the planning strategy
- Scenarios like ML-based Perception may exhibit non-gaussian, state dependent output uncertainties.
- The state dependence property can be leveraged and integrated into planning

- In our work, we establish a theory for set-valued observability analysis and design a metric to evaluate quality of state estimation.
- This metric can be used in an optimization problem to improve estimation in a planning task

Problem Setup

- General Non-linear System with set- valued uncertainty.

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t),$$

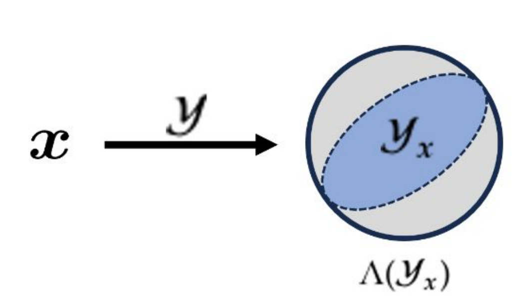
$$\boldsymbol{y}_t = h(\boldsymbol{x}_t),$$

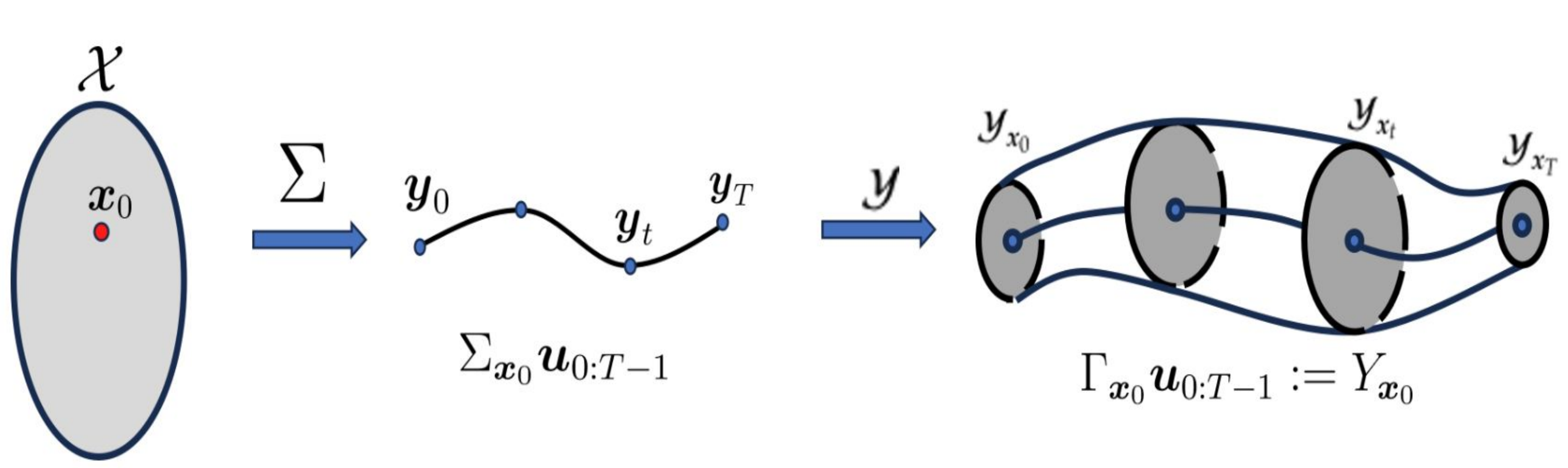
$$\boldsymbol{y}_t' = h(\boldsymbol{x}_t) + \boldsymbol{\varepsilon}_t,$$

- Uncertainties captured using Ellipsoids, can be propagated to predict output tubes

$$\mathcal{E}(\boldsymbol{x}_0, Q(\boldsymbol{x}_0)) = \{\boldsymbol{x} \mid (\boldsymbol{x}-\boldsymbol{x}_0)^\top Q(\boldsymbol{x}_0)^{-1}(\boldsymbol{x}-\boldsymbol{x}_0) \leq 1\}$$

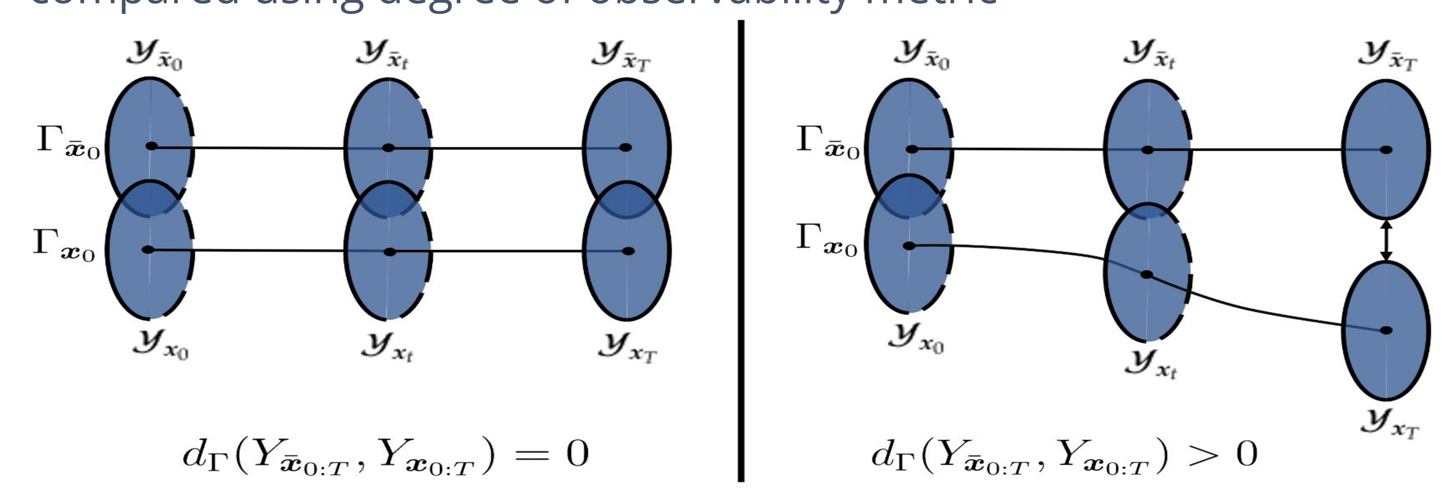
$$\mathcal{Y}_{\boldsymbol{x}_0} := \mathcal{E}(\boldsymbol{y}_0, Q(\boldsymbol{x}_0)),$$





Observability based metrics

- Trajectories with unobservable and observable output maps can be compared using degree of observability metric



- Optimization problem can be posed for trajectory planning where we maximize this metric::

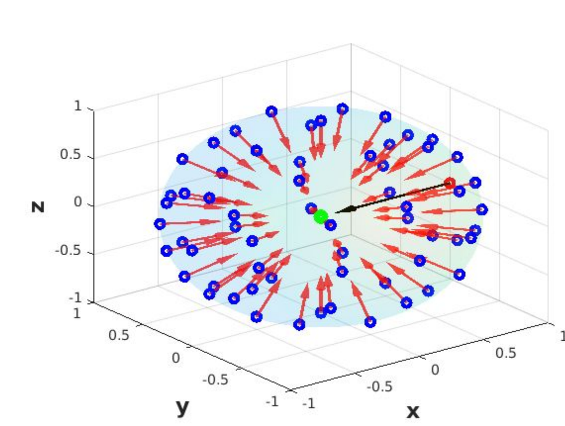
$$\min_{\boldsymbol{u}_{0:T-1}} \left(\sum_{t=0}^{T} c(\boldsymbol{x}_t, \boldsymbol{u}_t)\right) - \lambda_{\text{obs}} D_O^\ell(Y_{\boldsymbol{x}_{0:T}})$$

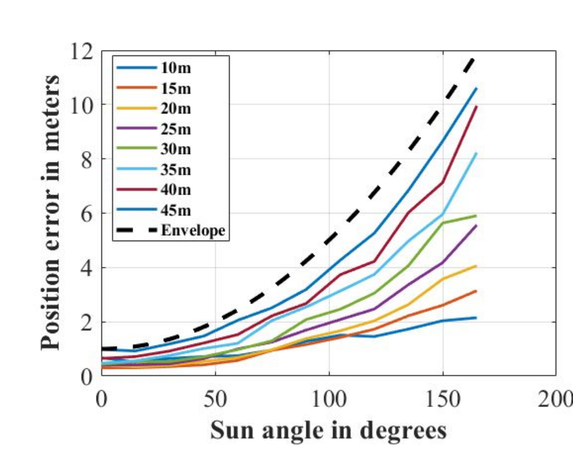
$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t, \boldsymbol{u}_t) \quad \forall t = 0, \cdots, T-1,$$

- This non-convex problem can be solved using sequential programming approaches.

Theory

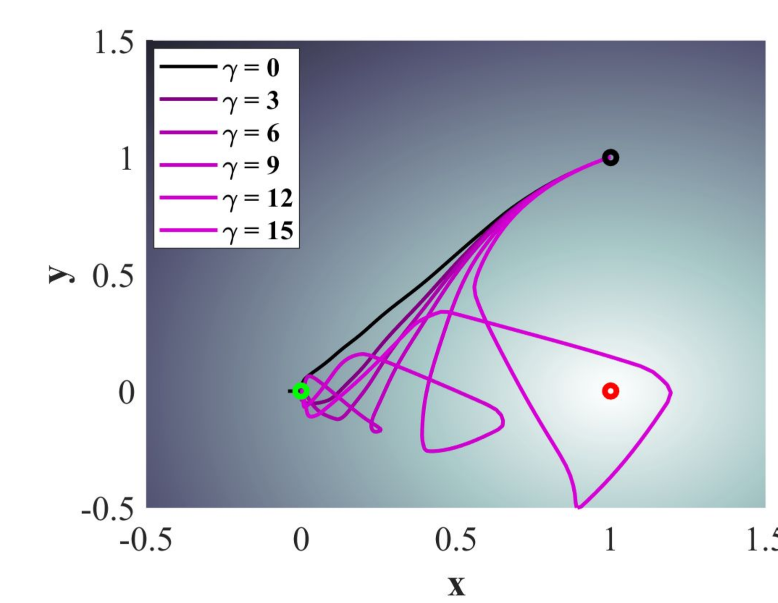
- **Assumption :** The uncertainty size is locally convex wrt state around a nominal trajectory.
- **Assumption :** System is locally linearizable
- If initial local perturbations to trajectories can be distinguished then, system is locally, finite horizon weakly observable
- Now the optimization problem becomes a conditioning problem and we formulate a metric called the degree of observability.
- A conservative lower bound of this metric is maximized in the optimization problem.
- For practical application validation experiments are required where upper bounding ellipsoids are generated for local uncertainty sizes

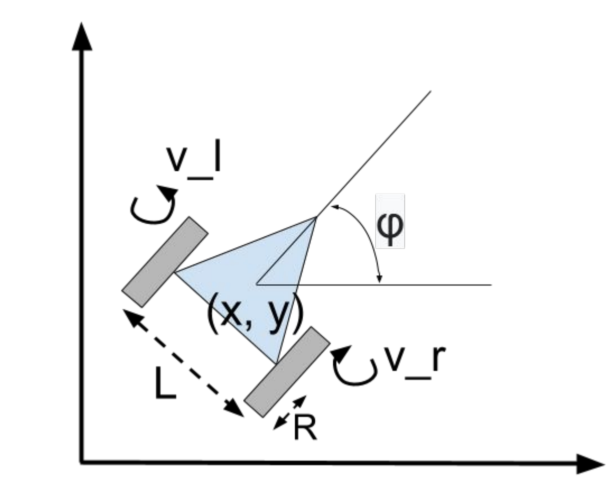


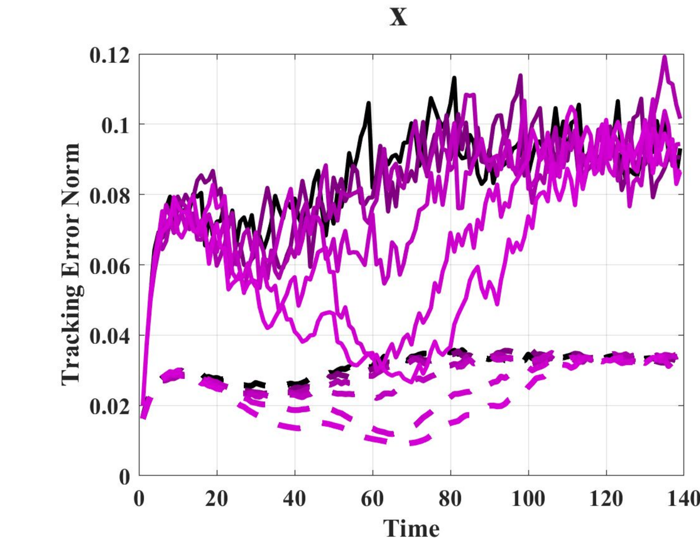


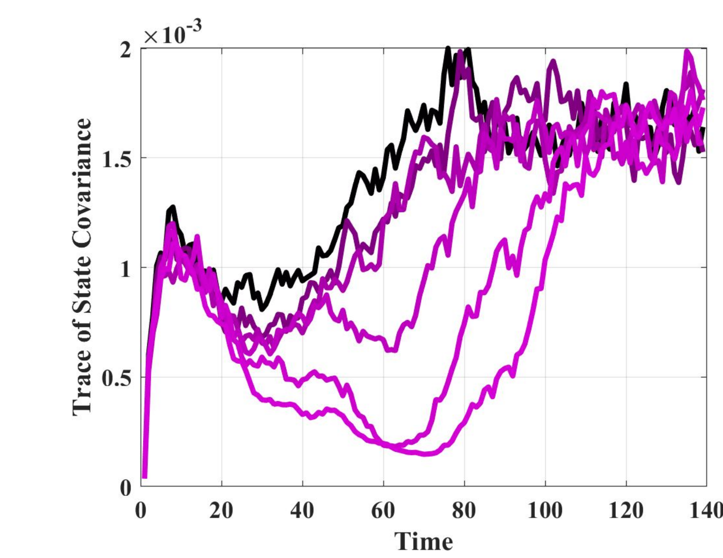
Case Study

- Double Integrator system navigating through a dark zone

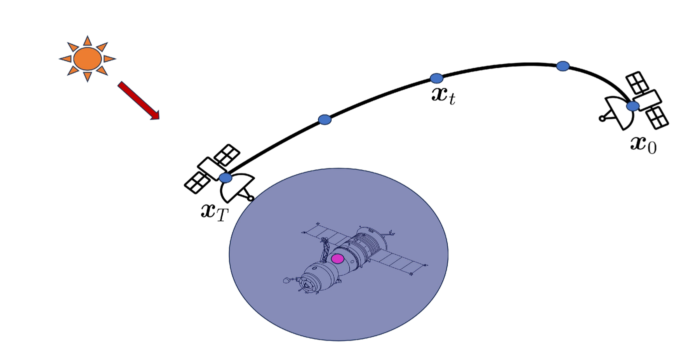


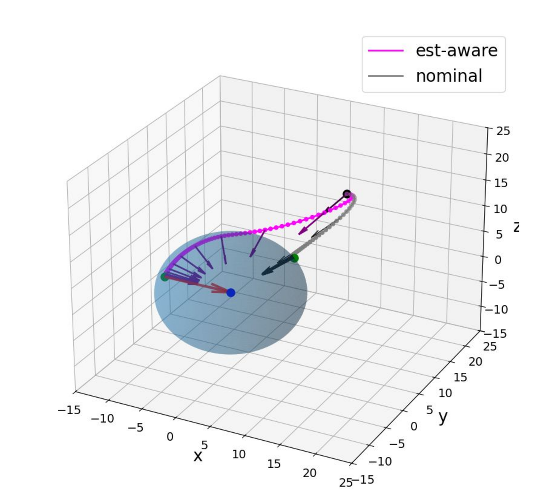


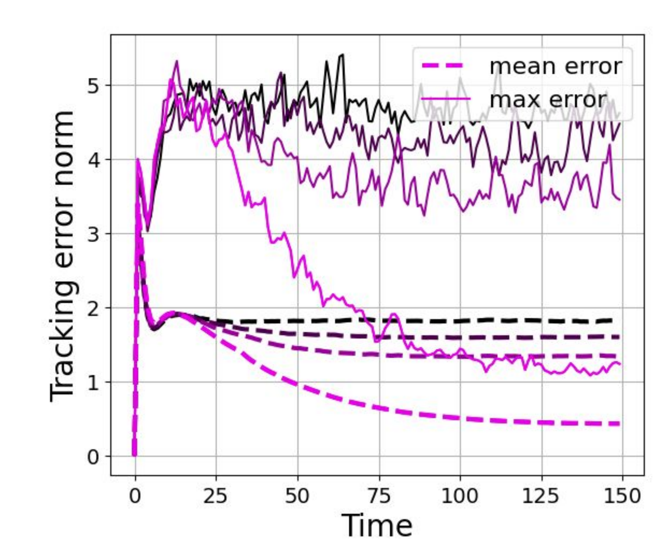


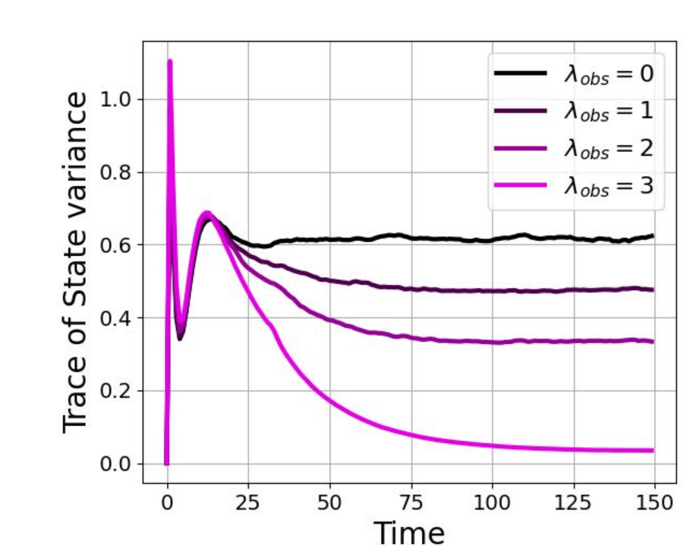


- Satellite tracking problem with ML-based uncertainties









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