QOCO: A Quadratic Objective Conic Optimizer with Custom Solver Generation

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Background

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{2}x^\top Px + c^\top x \\ \text{subject to} \quad & Gx \preceq_{\mathcal{K}} h \\ & Ax = b, \end{aligned}$$

- Quadratic objective second-order cone programs commonly arise in optimal control and trajectory optimization and must be solved in real-time
- Typically, the sparsity structure of the matrices P, A, and G are known beforehand
- When the sparsity structure is known, it is possible to generate a solver that uses custom linear algebra to exploit the sparsity structure and reduce solve-time

Limitations of Existing Software

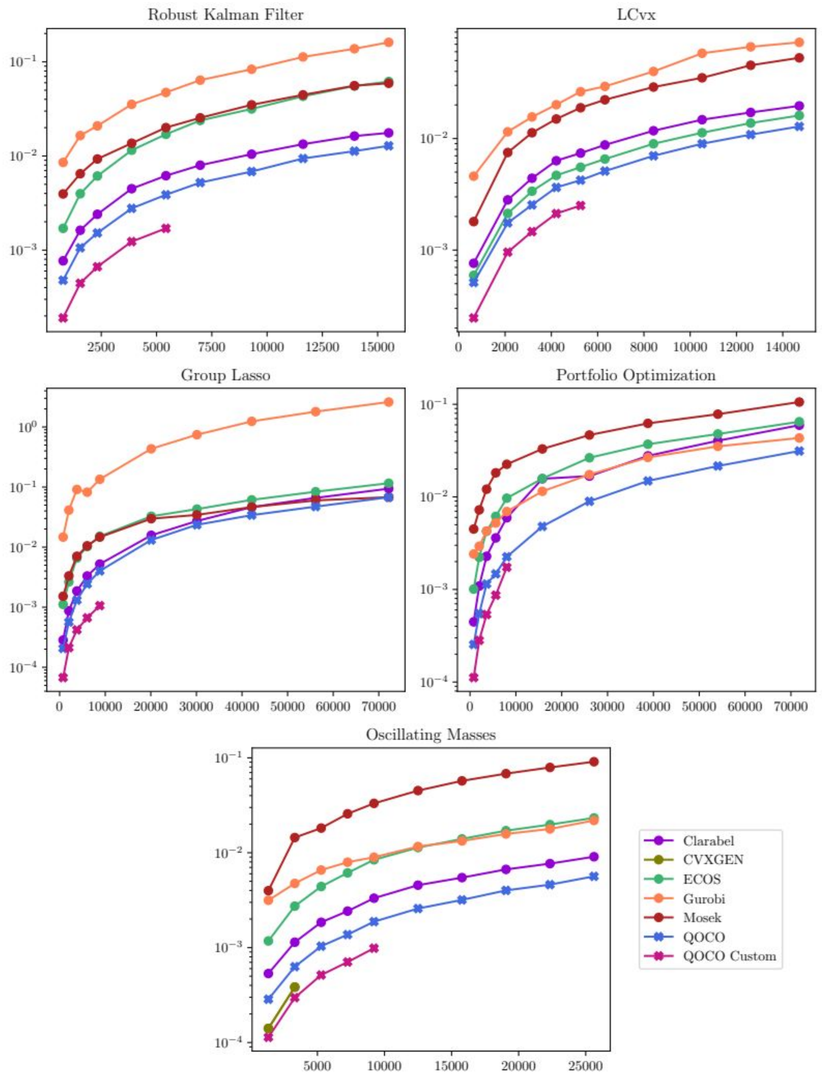
- Clarabel is an open-source solver for quadratic objective SOCPs, but it is written in Rust rather than C making it more challenging to use for legacy systems and is not a custom solver generator
- CVXGEN and BSOCP are custom solver generators for quadratic programs and linear objective SOCPs respectively, however neither can handle quadratic objective SOCPs, neither are open-source, and neither can be used with modern modeling languages such as CVXPY

Contributions

- In this work, we develop:
- **QOCO,** an open-source C-based solver for quadratic objective SOCP
- **QOCOGEN,** an open-source custom solver generator for quadratic objective SOCPs
- QOCO is faster and more robust than many commonly used solvers.
- Solvers generated by QOCOGEN are significantly faster than QOCO and are free of dynamic memory allocation making them an attractive option for real-time optimization on resource-constrained embedded systems
- Both solvers are easy to use as they can be called from CVXPY/CVXPYgen

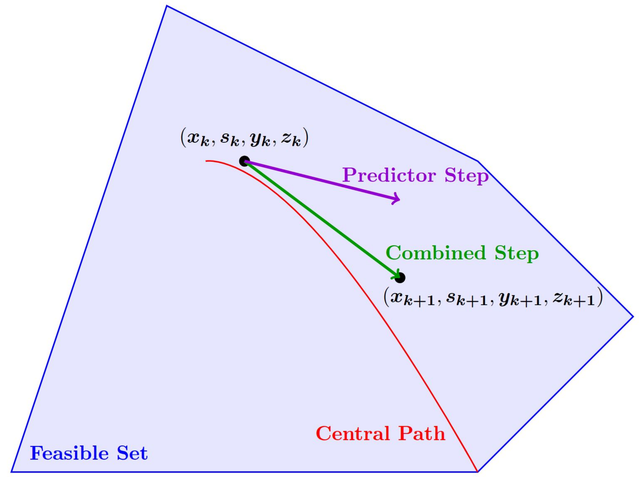
Numeric results

- We compared our solvers against a variety of open-source and commercial solvers
- The plot below depicts solve-time in seconds vs problem size for five optimization problem from various domains



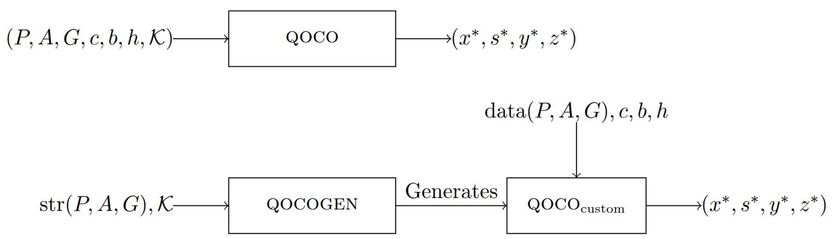
Primal-Dual Interior Point Method

- QOCO implements a primal-dual interior point method due to its robustness to ill-conditioning and good practical performance



Customized Linear Algebra

- In the primal-dual interior point method, the main computational bottleneck is the factorization of the KKT system
- Sparse linear algebra has extra overhead to locate nonzero elements and their positions in the matrix, since it must handle arbitrary sparsity patterns
- Customized linear algebra hard-codes the exact operations and memory accesses needed resulting in higher performance



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