Trace – a game-changing discovery

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Topics

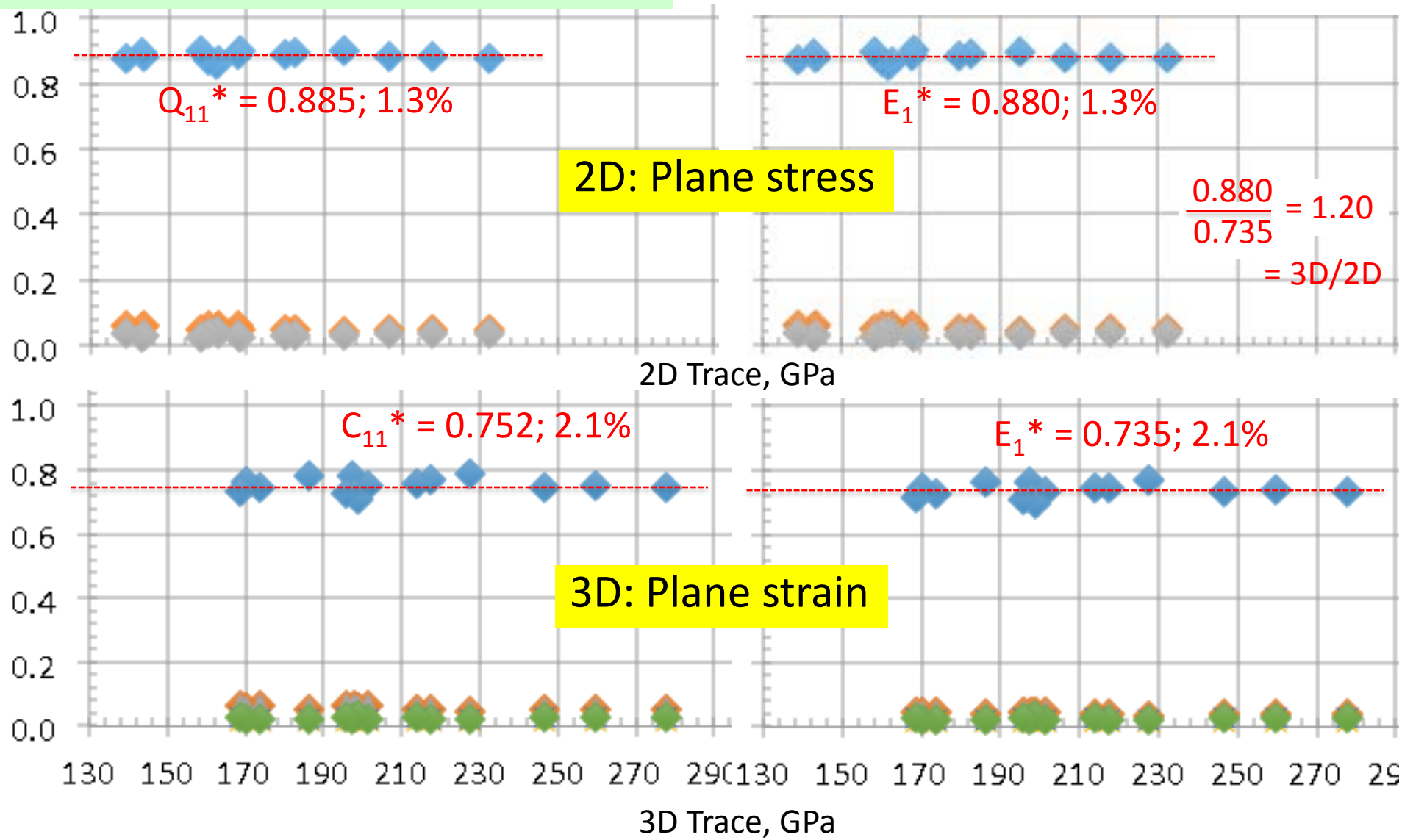
- Why trace? The one and only unifying constant
- Theory of trace – simple; easy to understand
- C-Ply by Stanford/Chomarat – bi-angle NCF
- Homogenization – optimizable; high-speed ATL
- Unit circle failure criterion
- Trace-based sizing method – power of C-Ply
- Conclusions

How Many Data Points?

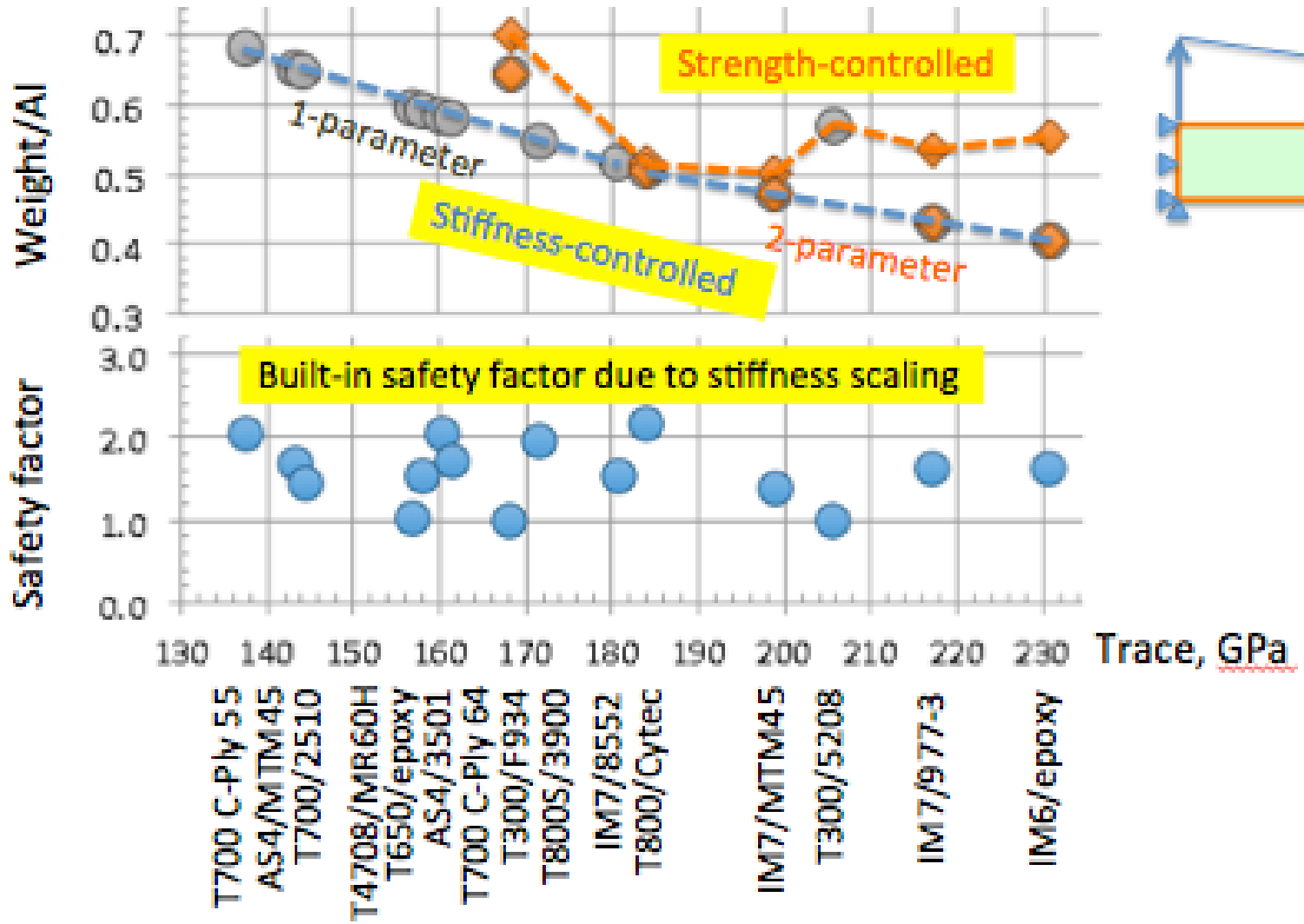
- In netting analysis that is applied to pressure vessels and pipes, fiber strength is the one and only design validation calculated from burst pressure: the answer is one
- Carbon fibers are known to be anisotropic but we know only the longitudinal fiber stiffness and strength: the answer is one as before, plus one for stiffness
- With trace, laminates become universal

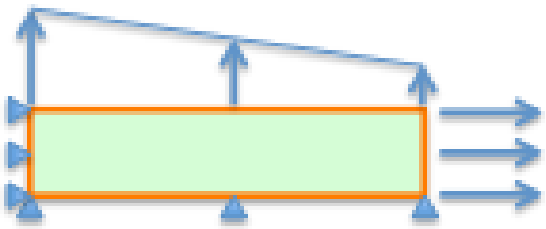
Dominant “11” Component in 2- & 3-D

Trace-normalized stiffness: Universal [0]



Material Selection for Weight Savings

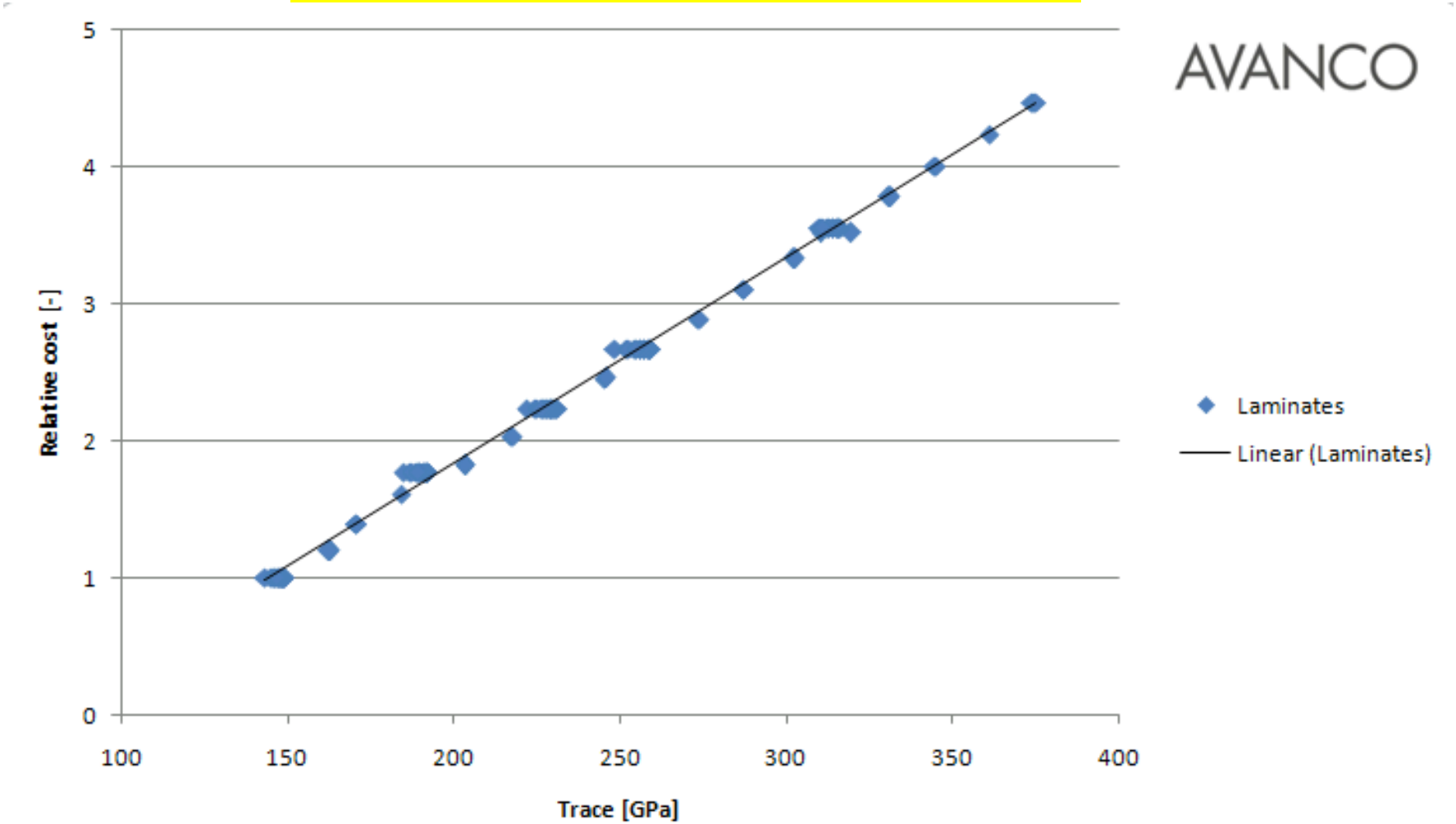




Relative Cost of Hybrid Laminates

Linear correlation with trace, but not with E_x or others

$$\text{Trace}^{\text{Hybrid}} = v_1\text{Trace}_1 + v_2\text{Trace}_2 + \ldots$$



Black Aluminum vs Trace: 15 CFRP

E_x = 0.88 Trace

Black aluminum vs Trace

Ply stiffness

E_y

ν_x

E_s

Ply strength

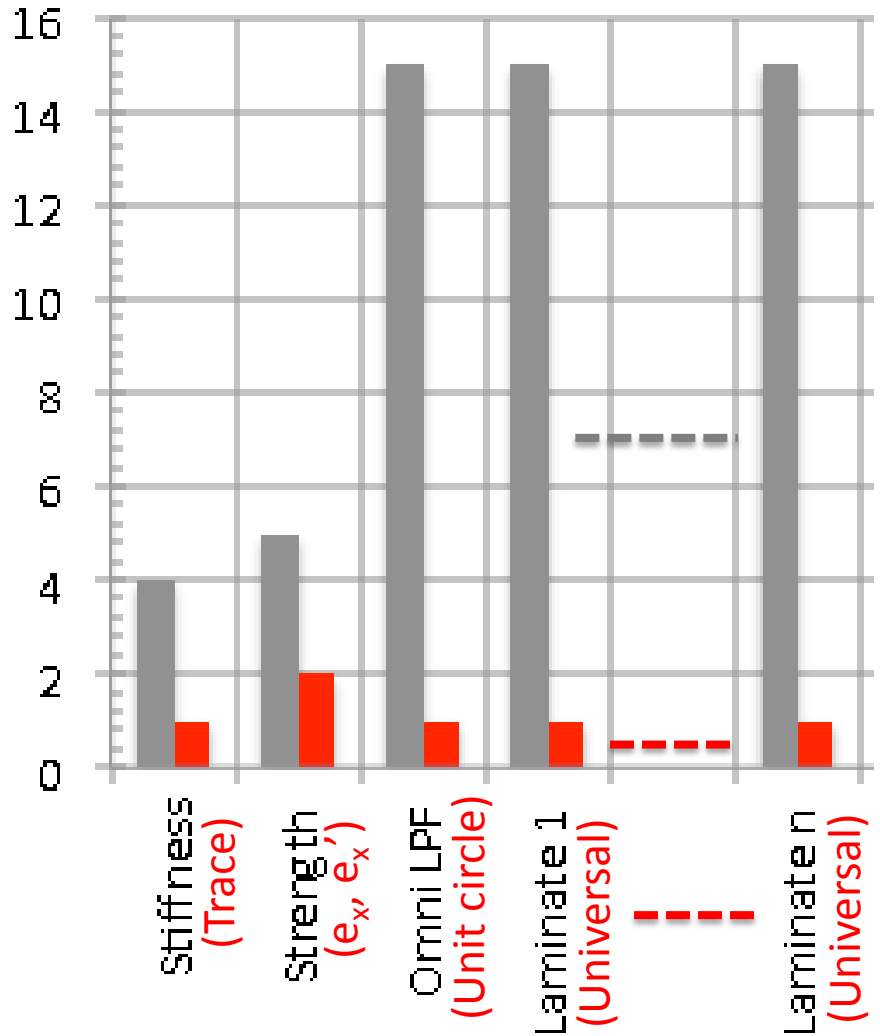
X

X'

Y

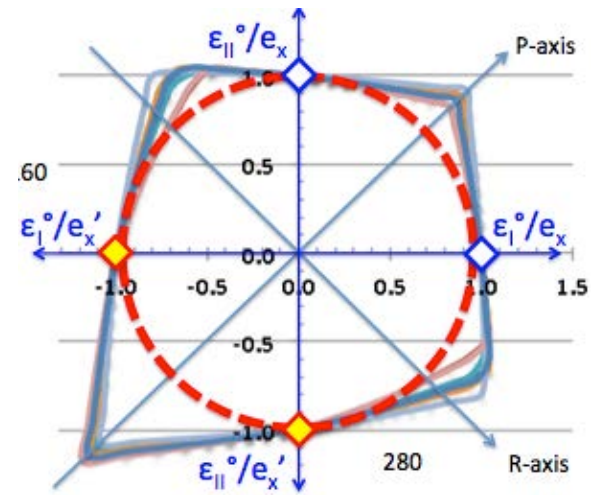
Y'

S

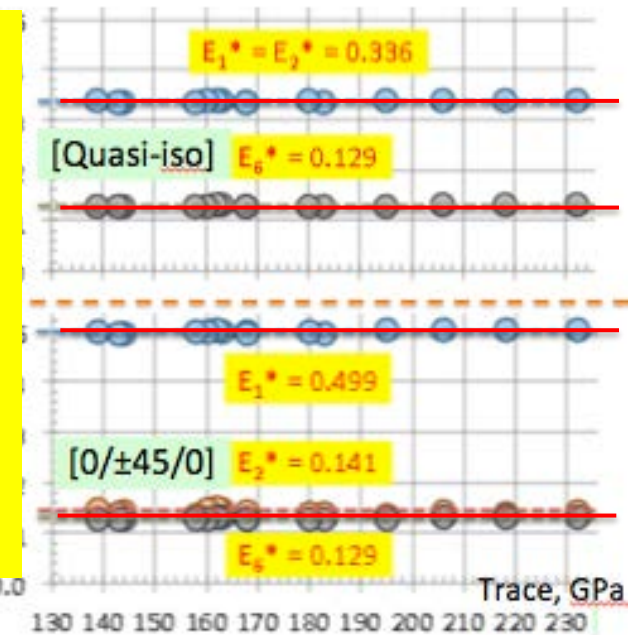


So much simpler: 4:1 5:2 15:1 15:1 ---- 15:1

Unit circle



Universal Laminates



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Tensors and Traces for Composites

- Stress components σ_i
- Strain components ε_i
- Stiffness: Q_{ij}, A_{ij}, D_{ij}, C_{ij},…
- Failure criterion in stress space: F_{ij}, F_i

  where $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$
- Failure criterion in strain space: G_{ij}, G_i

  where $G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$

  $G_{ij} = Q_{ik}Q_{jl}F_{kl}$. $G_i = Q_{ij}F_j$

$\mathrm{Tr}\,[\sigma] = \sigma_1 + \sigma_2 + \sigma_3 = \text{pressure}$

$\mathrm{Tr}\,[\varepsilon] = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \Delta\ \text{volume}$

2D

$\mathrm{Tr}\,[Q] = Q_{11} + Q_{22} + 2Q_{66}$

$= \mathrm{Tr}\,[A^*] = A_{11}^* + A_{22}^* + 2A_{66}^*$

$= \mathrm{Tr}\,[D^*] = D_{11}^* + D_{22}^* + 2D_{66}^*$

3D

$\mathrm{Tr}\,[C] = C_{11} + C_{22} + C_{33} + \ldots + 2C_{66}$

$\mathrm{Tr}\,[F] = F_{11} + F_{22} + F_{66}/2$

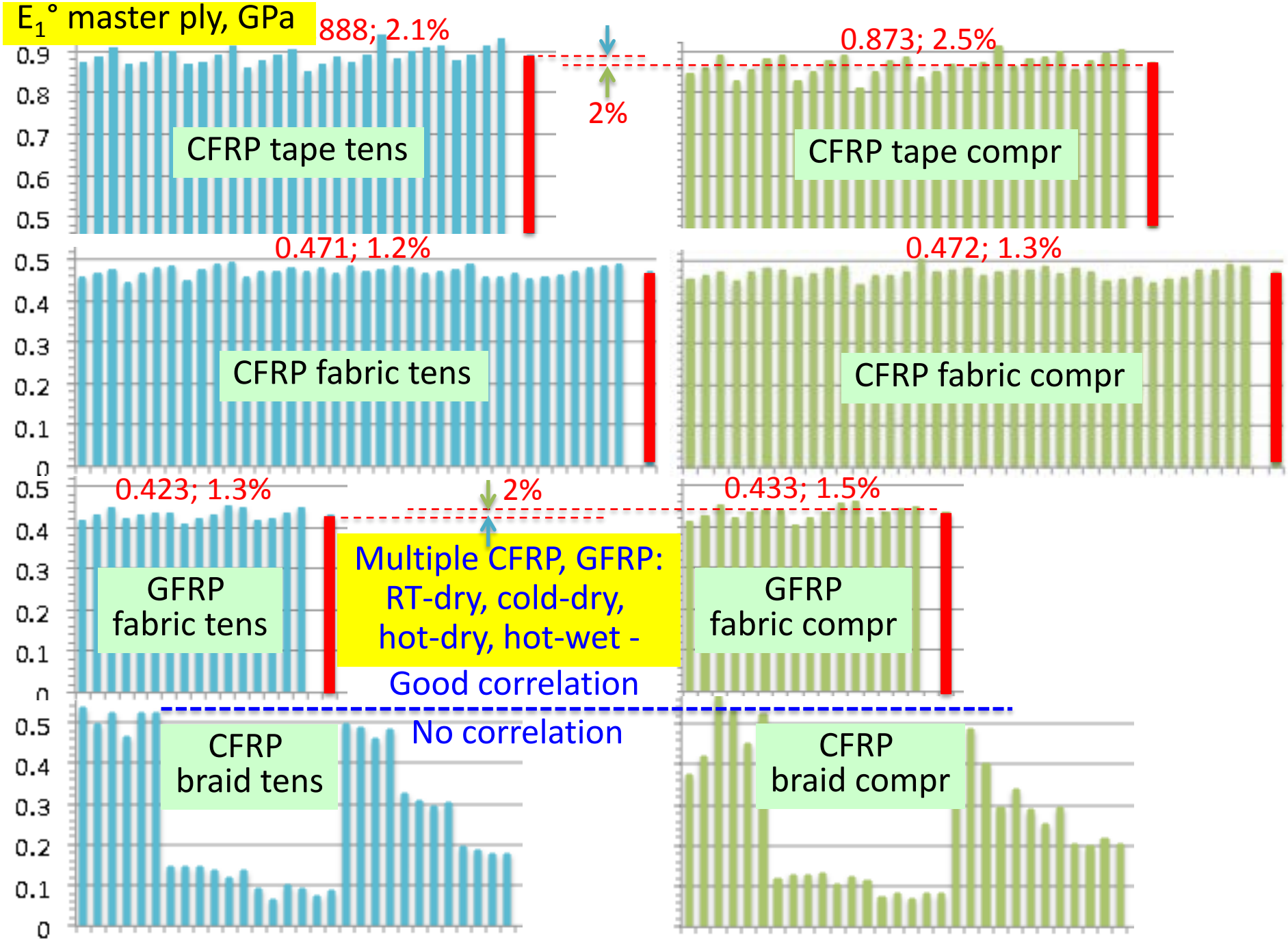
$\mathrm{Tr}\,\{F\} = F_1 + F_2$

$\mathrm{Tr}\,[G] = G_{11} + G_{22} + 2G_{66}$

$\mathrm{Tr}\,\{G\} = G_1 + G_2$

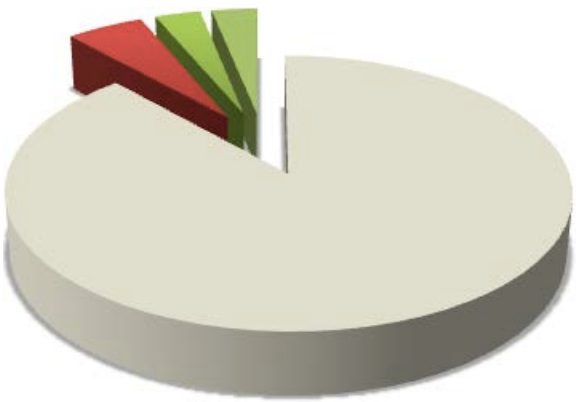
Evolution: Universal Constants [0] CFRP

	Ply engineering constants				Plane-stress stiff matrix				Trace	Trace-normalized		
15 Ply material	E_1	E_2	E_6	ν_{21}	Q_{11}	Q_{22}	Q_{21}	Q_{66}	Trace	E_1*	E_2*	E_6*
IM6/epoxy	203	11.2	8.4	0.32	204	11.3	3.6	8.4	232	0.874	0.048	0.036
IM7/977-3	191	9.9	7.8	0.35	192	10.0	3.5	7.8	218	0.877	0.046	0.036
T300/5208	181	10.3	7.2	0.28	182	10.3	2.9	7.2	206	0.877	0.050	0.035
IM7/MTM45	175	8.2	5.5	0.33	176	8.2	2.7	5.5	195	0.897	0.042	0.028
T800/Cytec	162	9.0	5.0	0.40	163	9.1	3.6	5.0	183	0.888	0.049	0.027
IM7/8552	159	9.0	5.5	0.32	160	9.0	2.9	5.5	180	0.884	0.050	0.031
T800S/3900	151	8.2	4.0	0.33	152	8.2	2.7	4.0	168	0.898	0.049	0.024
T300/F934	148	9.7	4.6	0.30	149	9.7	2.9	4.6	168	0.883	0.058	0.027
T700 C-Ply 64	141	9.3	5.8	0.30	142	9.4	2.8	5.8	163	0.866	0.057	0.036
AS4/H3501	138	9.0	7.1	0.30	139	9.0	2.7	7.1	162	0.852	0.055	0.044
T650/epoxy	139	9.4	5.5	0.32	140	9.5	3.0	5.5	160	0.866	0.059	0.034
T4708/MR60H	142	7.7	3.8	0.34	143	7.8	2.6	3.8	158	0.897	0.049	0.024
T700/2510	126	8.4	4.2	0.31	127	8.5	2.6	4.2	144	0.877	0.058	0.029
AS4/MTM45	127	7.9	3.6	0.30	128	8.0	2.4	3.6	143	0.889	0.055	0.025
T700 C-Ply 55	121	8.0	4.7	0.30	122	8.0	2.4	4.7	139	0.869	0.057	0.034
					Trace = $Q_{11} + Q_{22} + 2Q_{66}$							
Average = Universal				0.320					1.000	0.880	0.052	0.031
Std dev				0.029						0.013	0.005	0.006


E1° master ply, GPa
888; 2.1%
0.873; 2.5%
2%
CFRP tape tens
CFRP tape compr
0.471; 1.2%
0.472; 1.3%
CFRP fabric tens
CFRP fabric compr
0.423; 1.3%
2%
0.433; 1.5%
GFRP fabric tens
Multiple CFRP, GFRP: RT-dry, cold-dry, hot-dry, hot-wet -
GFRP fabric compr
Good correlation
No correlation
CFRP braid tens
CFRP braid compr

Composition of Trace: Carbon, Kev, Glass

(88, 5, 3)%

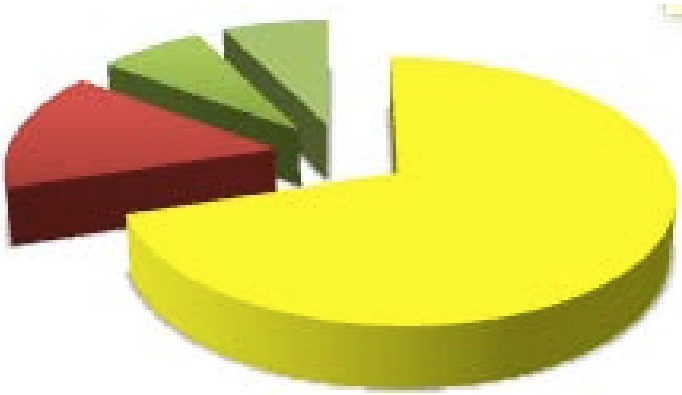


Carbon/epoxy

(88, 6, 3)%

Kevlar 49/epoxy

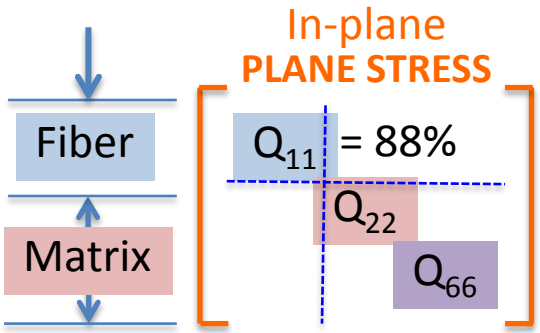
(70, 15, 7)%

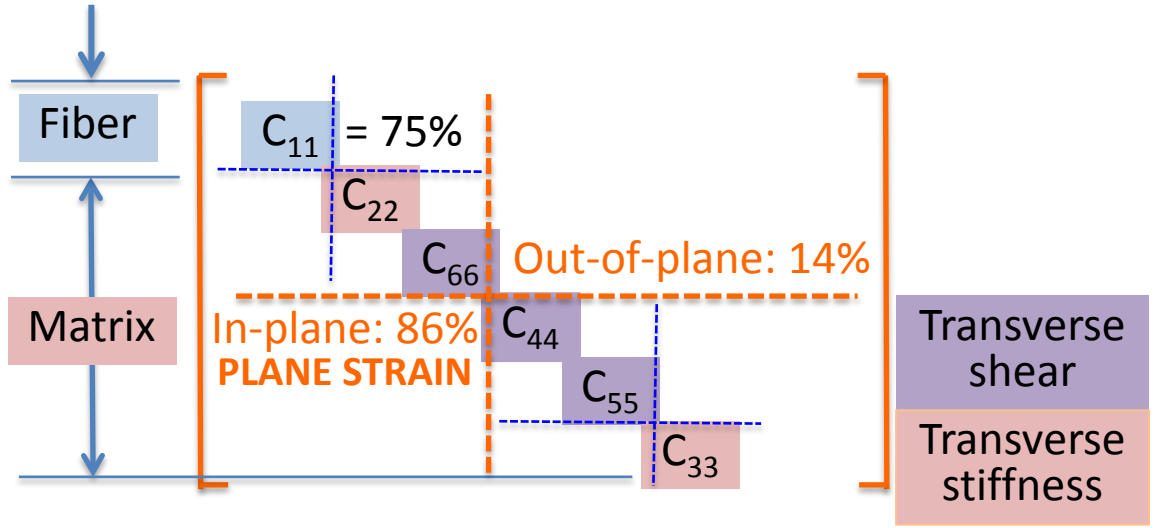


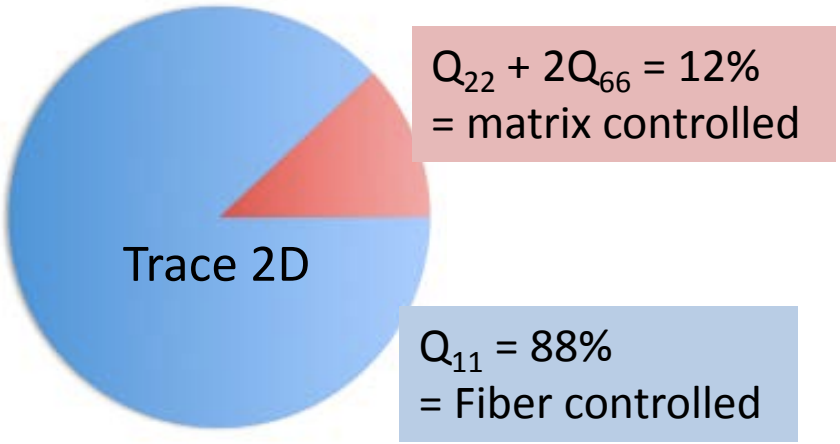
E-glass/epoxy

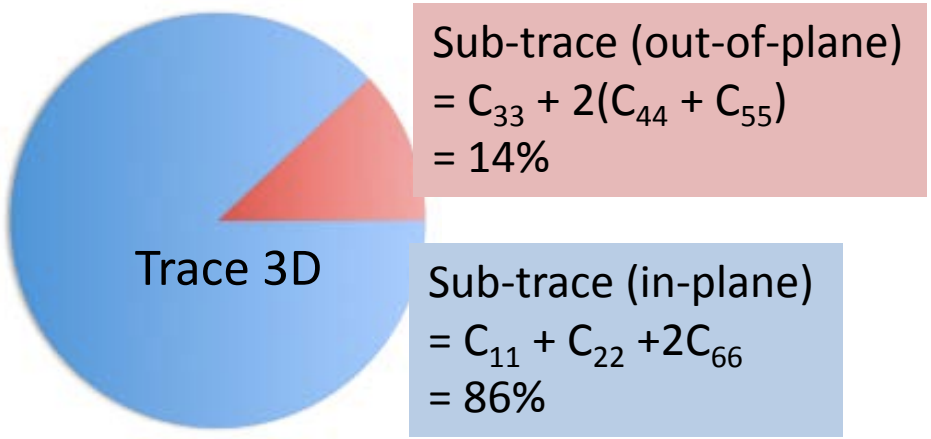
Ply material	Q_{xx}*	Q_{yy}*	Q_{ss}*	Q_{xy}*	Trace
Average CFRP	0.88	0.05	0.03	0.02	1.000
Kevlar/epoxy	0.88	0.06	0.03	0.03	1.000
E-glass/epoxy	0.70	0.15	0.07	0.04	1.000

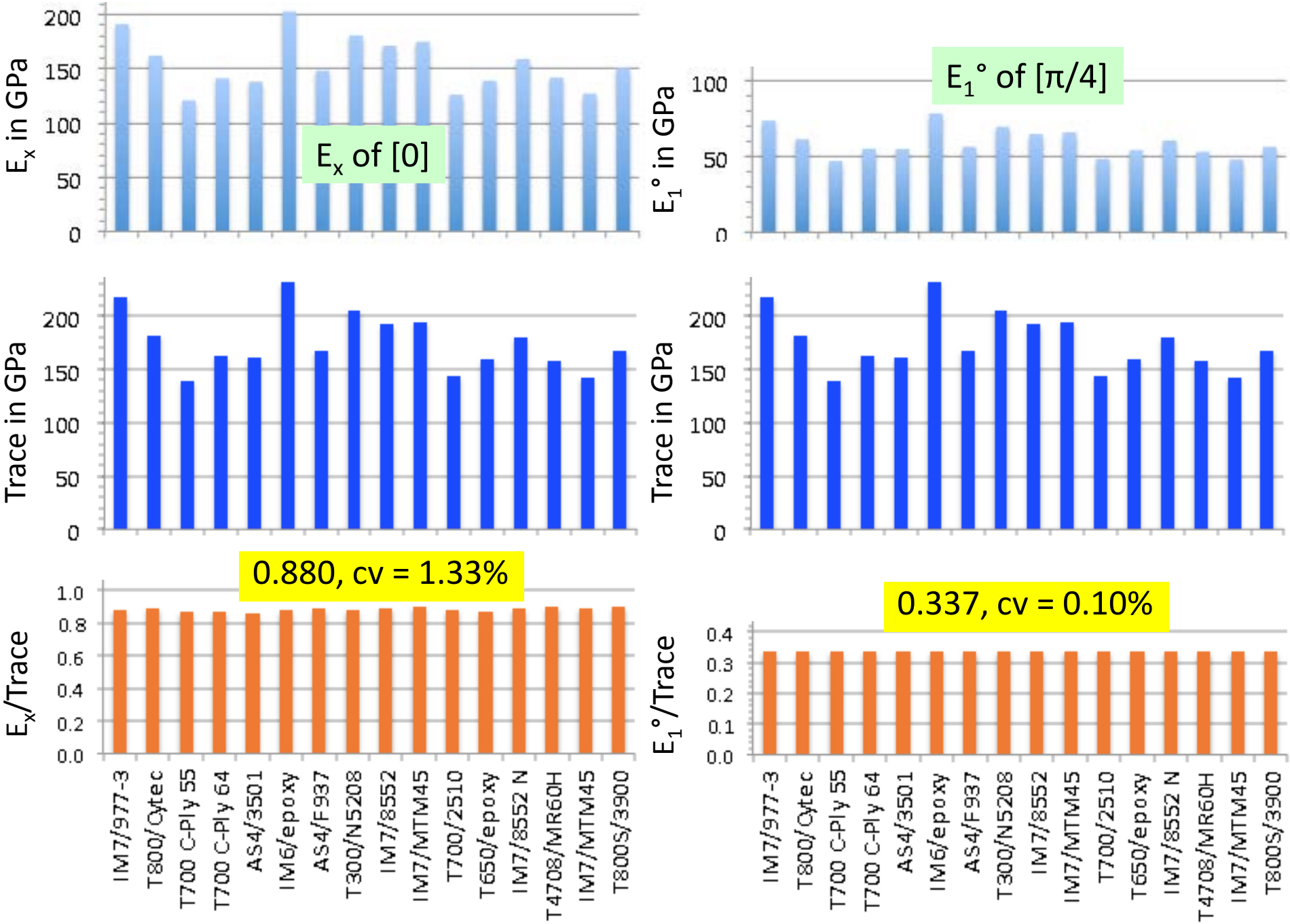
Universal ply: Transversely isotropic [0]





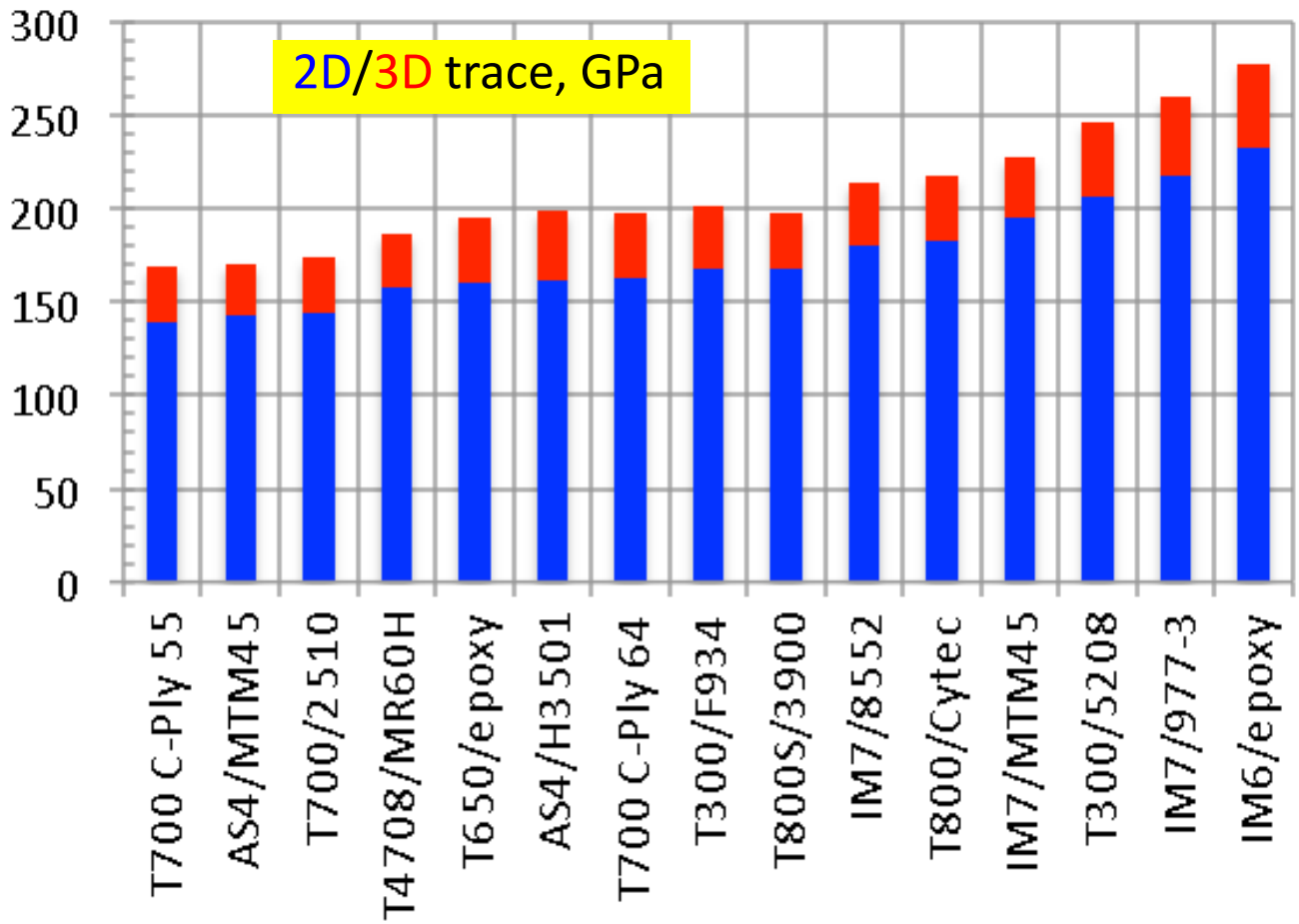





E_x in GPa
E_x of [0]
$E_1°$ in GPa
$E_1°$ of [π/4]
Trace in GPa
Trace in GPa
E_x/Trace
0.880, cv = 1.33%
$E_1°$/Trace
0.337, cv = 0.10%
IM7/977-3
T800/Cytec
T700 C-Ply 55
T700 C-Ply 64
AS4/3501
IM6/epoxy
AS4/F937
T300/N5208
IM7/8552
IM7/MTM45
T700/2510
T650/epoxy
IM7/8552 N
T4708/MR60H
IM7/MTM45
T800S/3900

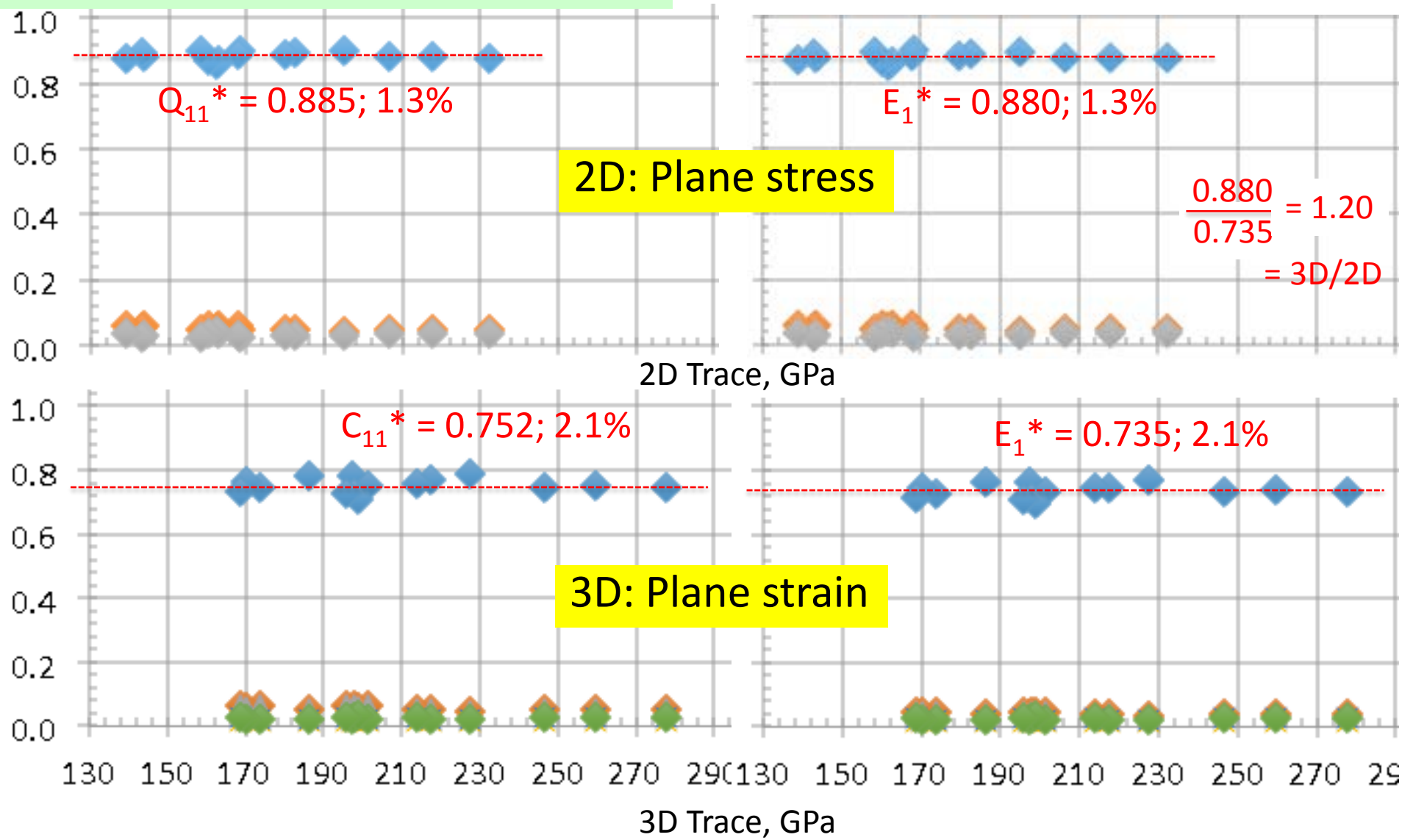
Ascending 2D and 3D Trace: CFRP

15 Ply material	2D Tr	3D Tr	3D/2D
T700 C-Ply 55	139	169	1.21
AS4/MTM45	143	170	1.19
T700/2510	144	174	1.21
T4708/MR60H	158	186	1.18
T650/epoxy	160	196	1.22
AS4/H3501	162	199	1.23
T700 C-Ply 64	163	198	1.22
T300/F934	168	201	1.20
T800S/3900	168	197	1.17
IM7/8552	180	214	1.19
T800/Cytec	183	217	1.19
IM7/MTM45	195	227	1.17
T300/5208	206	247	1.19
IM7/977-3	218	260	1.19
IM6/epoxy	232	278	1.20
Average			1.20
Std deviation			1.7%



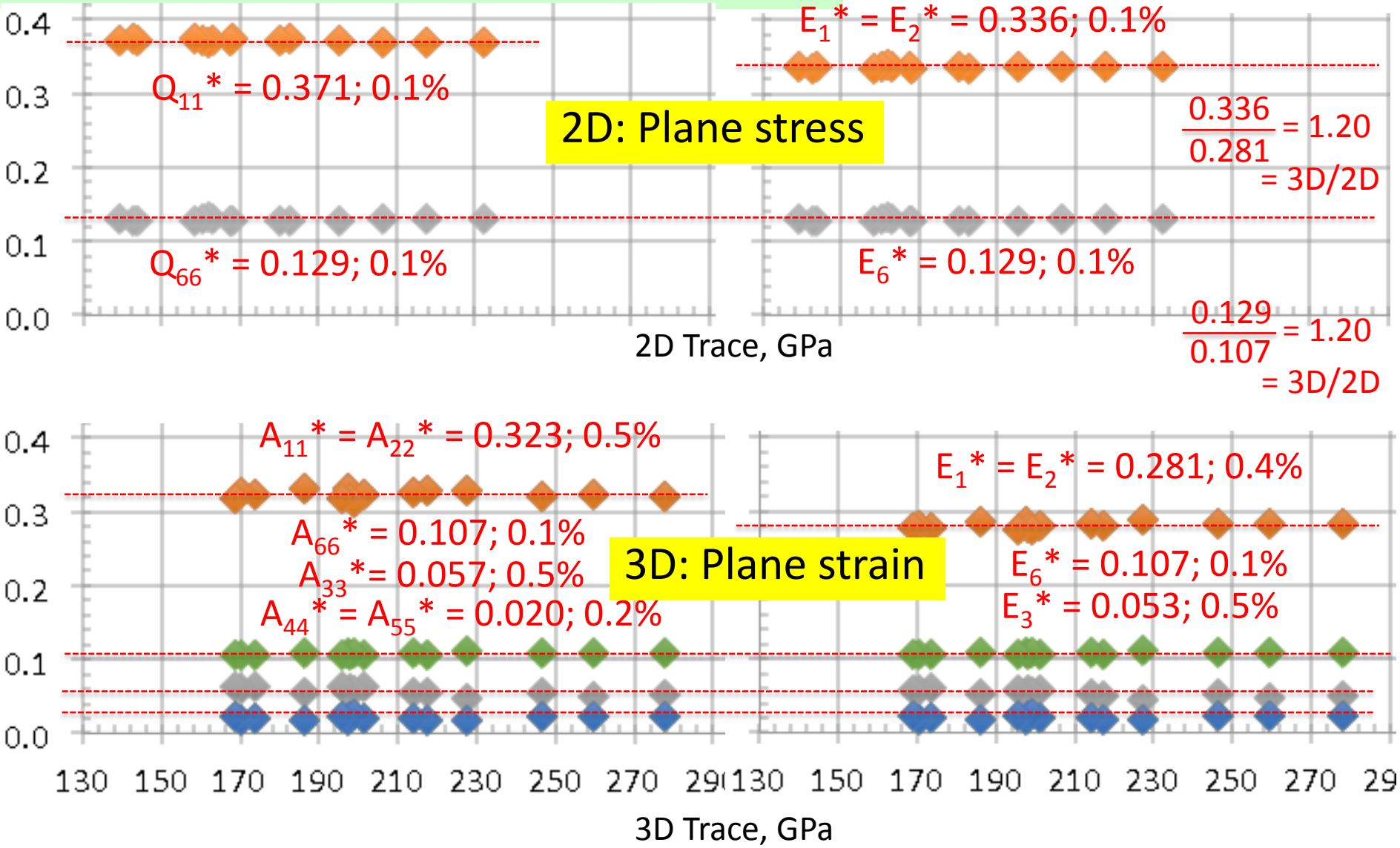
Dominant “11” Component in 2- & 3-D

Trace-normalized stiffness: Universal [0]

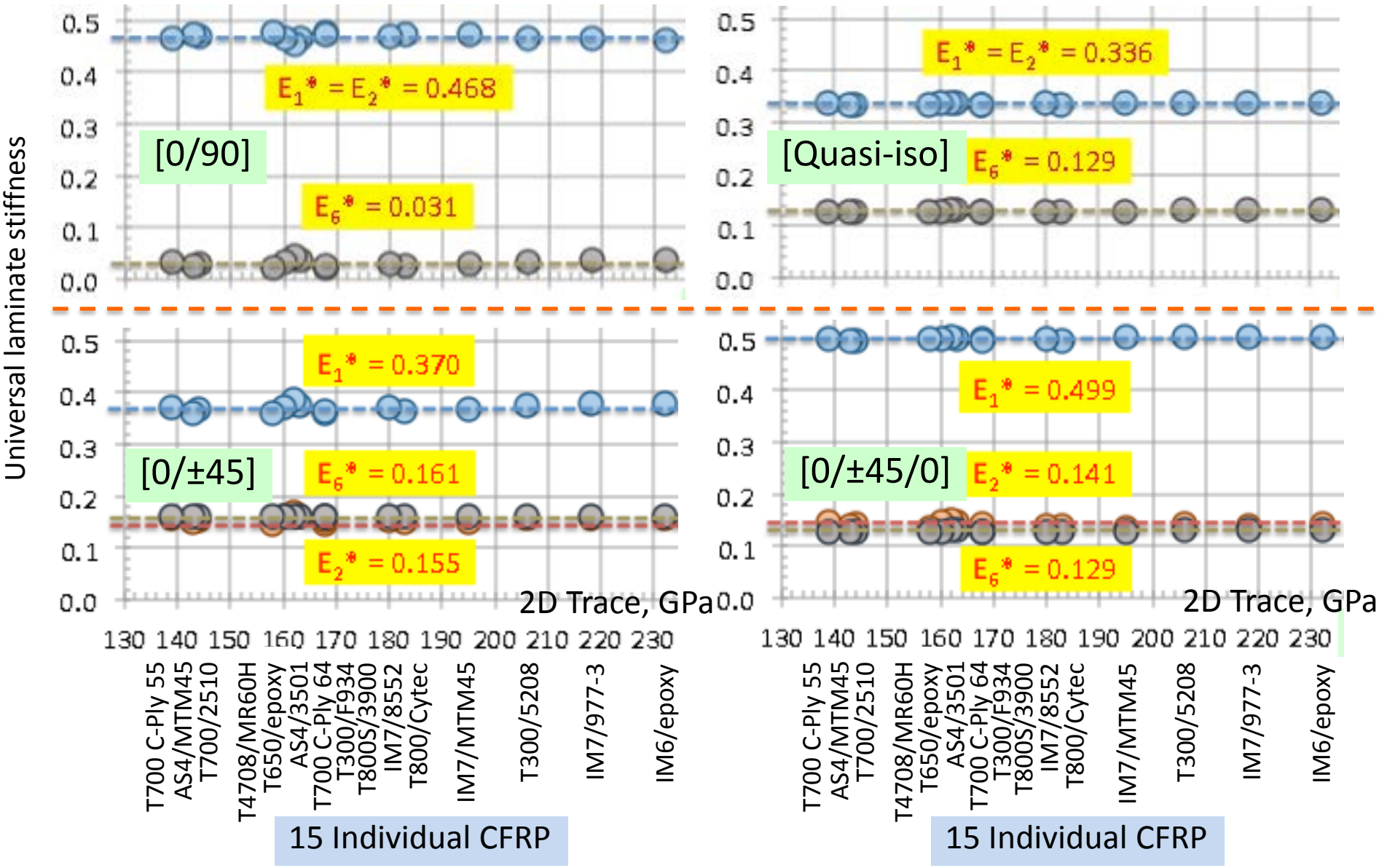


Dominant “11” Component in 2- & 3-D

Trace-normalized stiffness: Universal [π/3],[π/4], …



Components: 2D Universal Laminates

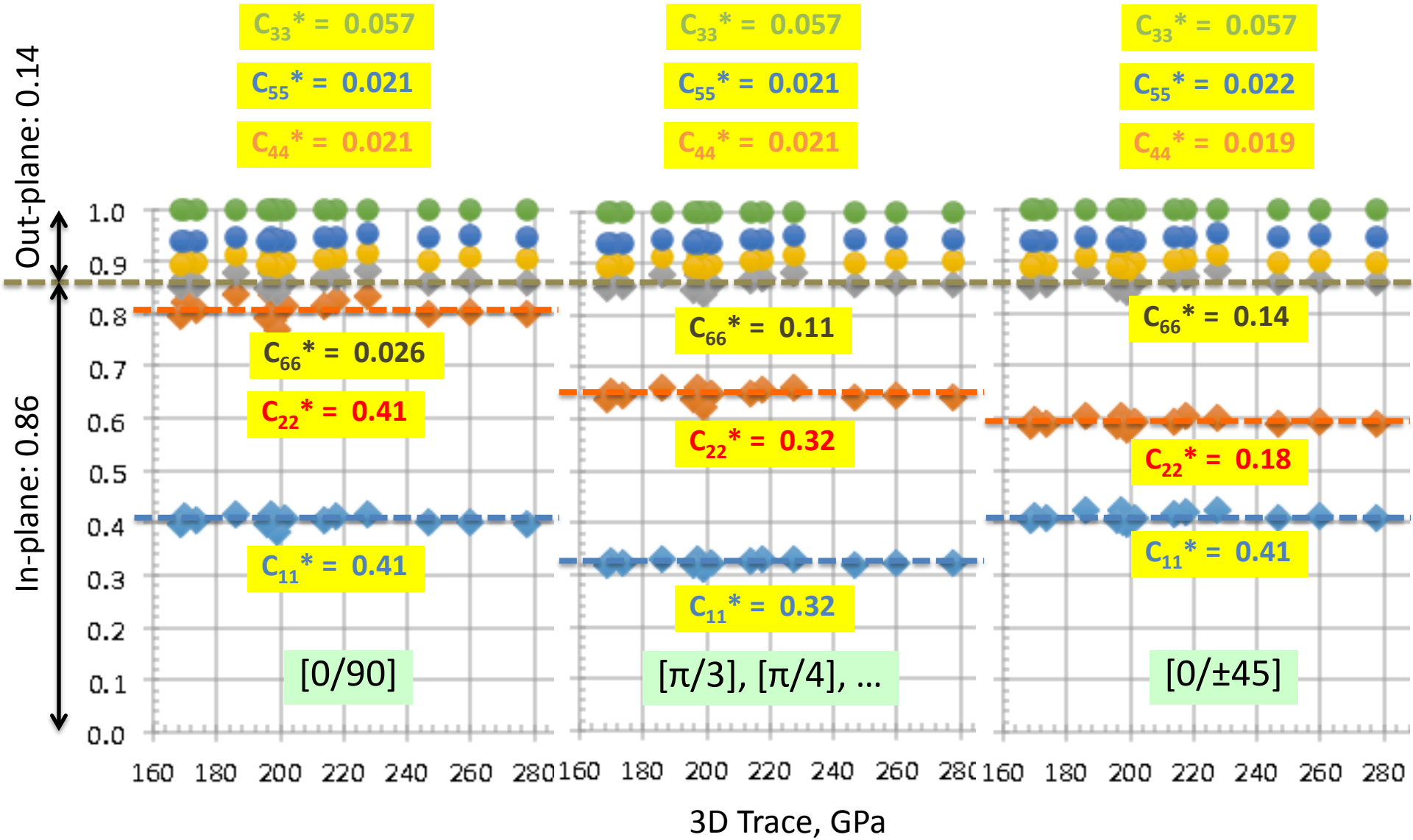


Universal Laminate Constants

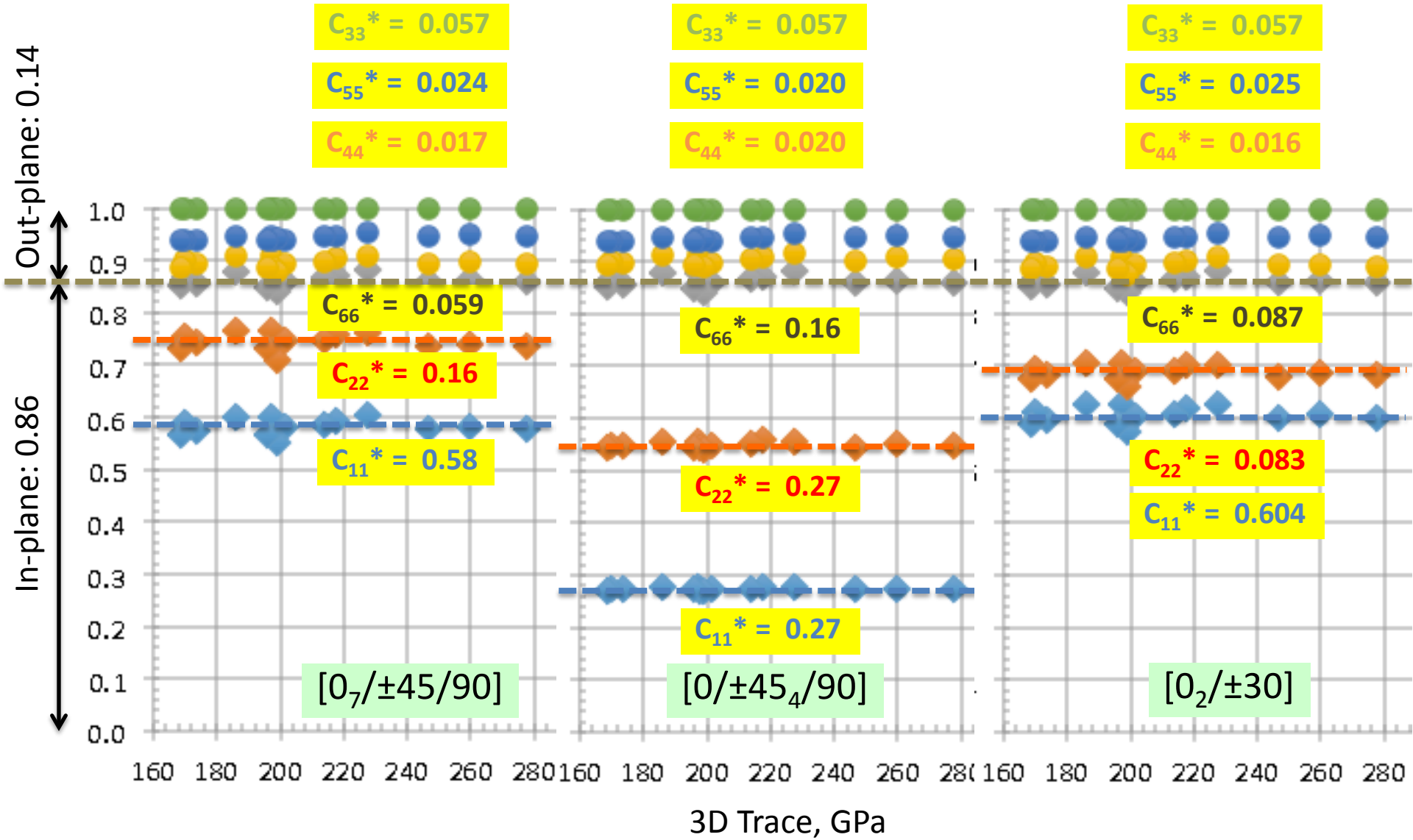
Generated from classical laminated plate theory, and universal [0] derived from average of 15 CFRP

		E_1*	E_2*	E_6*	ν_{21}	Trace
	Universal [0]	0.880	0.052	0.031	0.320	1.000
Universal laminates	[0/90]	0.468	0.468	0.031	0.036	1.000
	[π/4]	0.336	0.336	0.129	0.308	1.000
	$[0_7/\pm45/90]$	0.662	0.175	0.070	0.310	1.000
	$[0/\pm45_4/90]$	0.217	0.217	0.187	0.552	1.000
	[0/±45]	0.370	0.155	0.161	0.734	1.000
	[0/±45/0]	0.499	0.141	0.129	0.701	1.000
	[0/±30]	0.510	0.074	0.129	1.220	1.000
	[0/±30/0]	0.611	0.072	0.104	1.079	1.000
	[±12.5]	0.764	0.053	0.066	0.913	1.000

Components: 3D Universal Laminates



Components: 3D Universal Laminates



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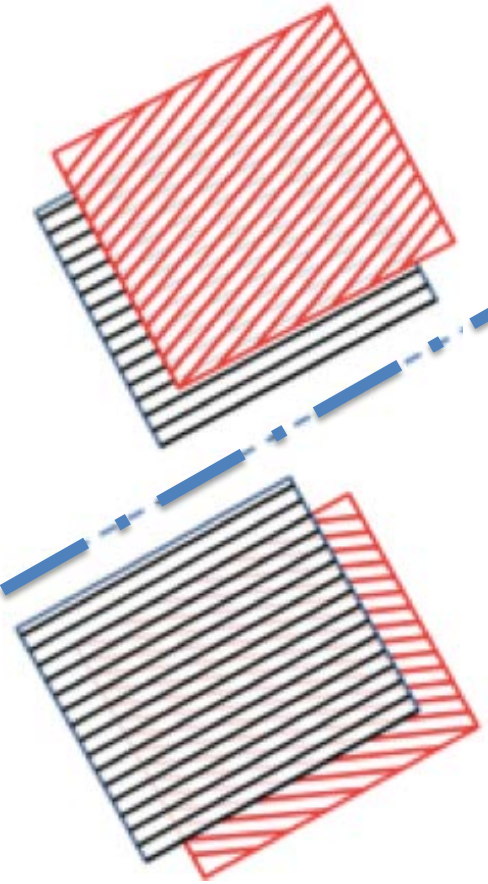
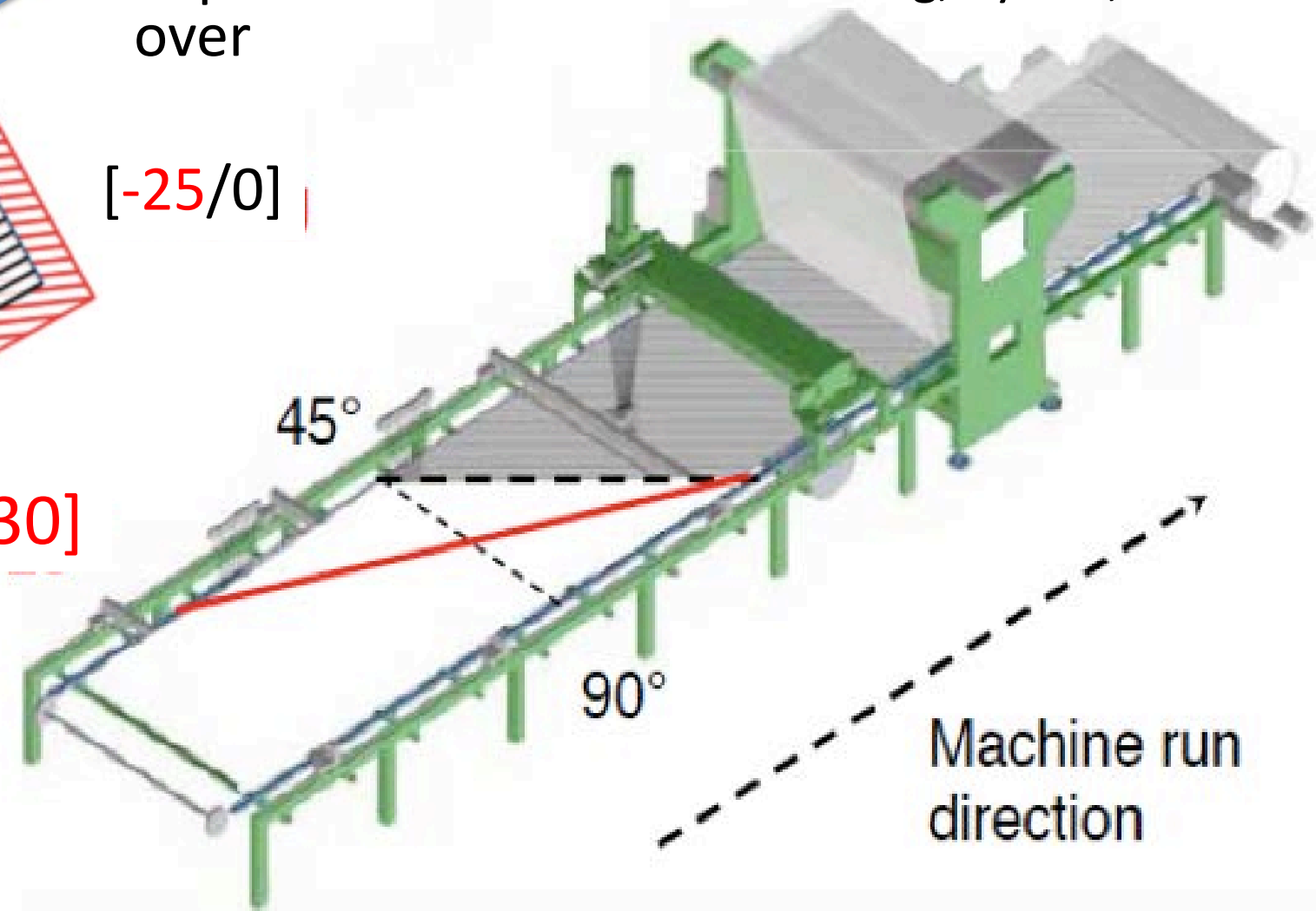
Unique NCF at Chomarat

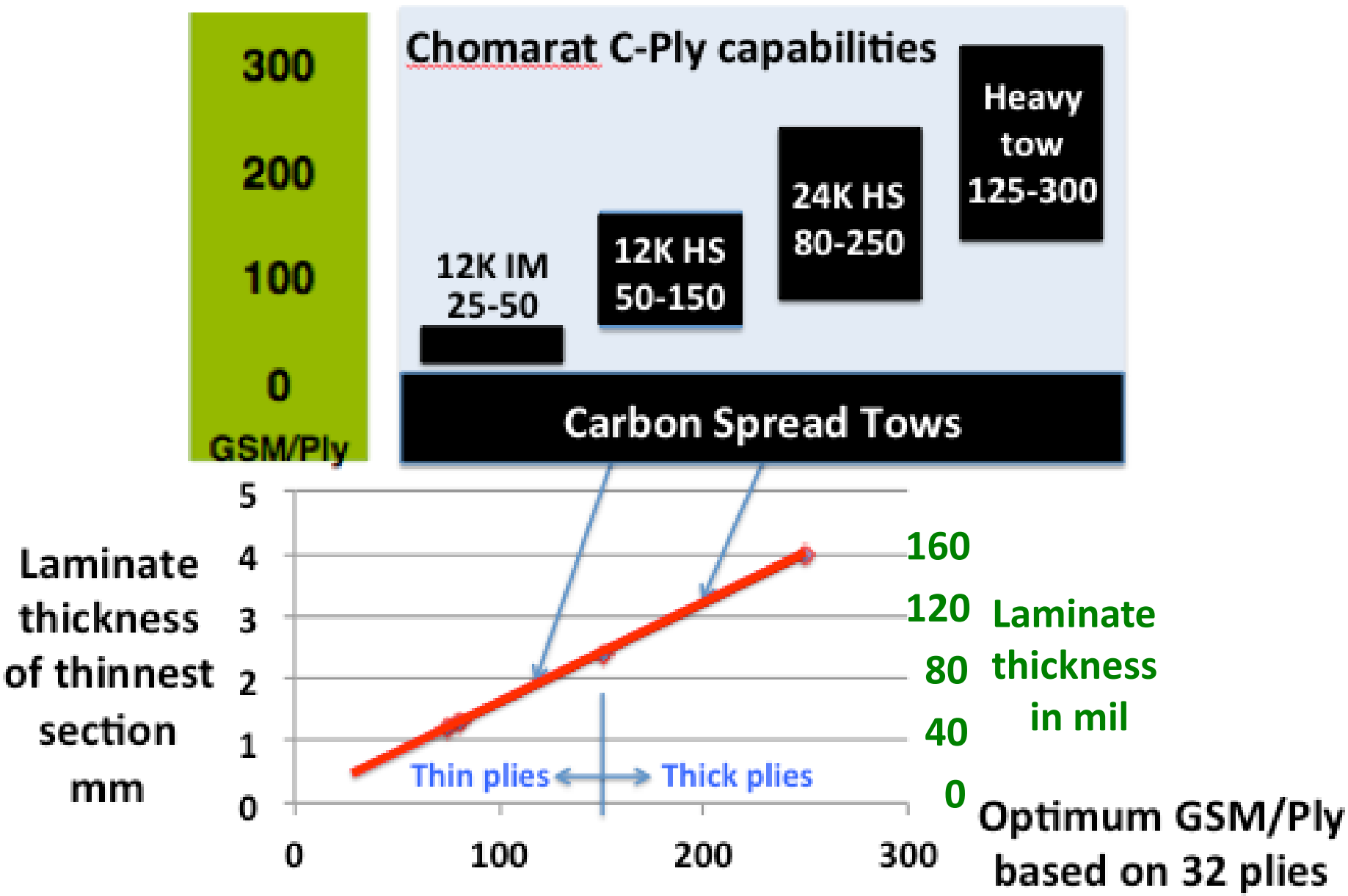
Shallow angles, wide range ply thicknesses with high quality from tow spreading, noninvasive stitching, hybrid, …



[20 ≤ φ ≤ 30]

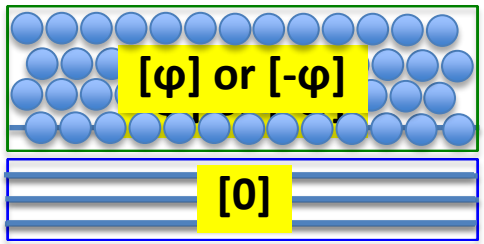


Wide-range GSM to Meet Requirement

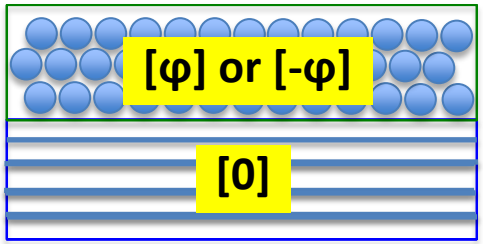


Thick-thin Combinations of C-Ply

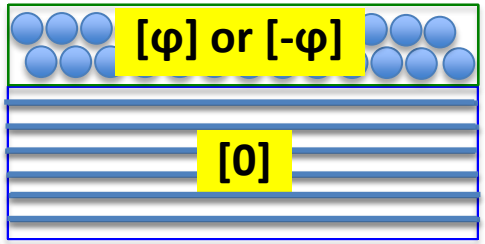
Thin [0]; thick [φ]



Thin [0]; thin [φ]

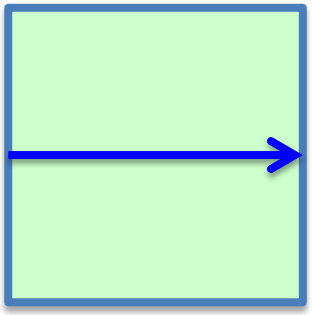


Thick [0]; thin [φ]



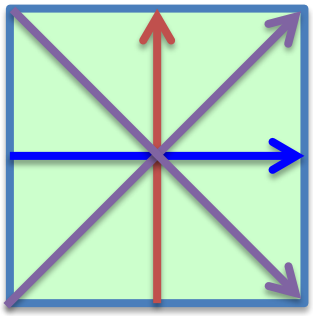
Black Alum

[0]



1-axis

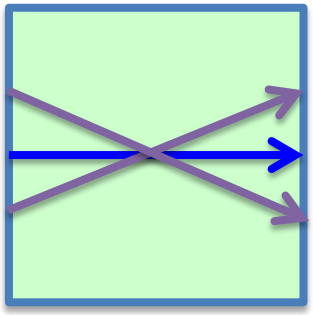
[0/±45/90]



4-axis

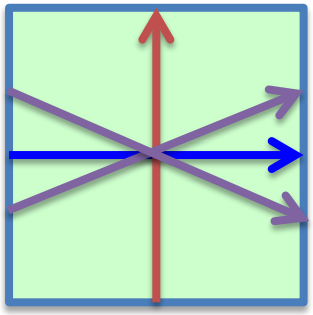
C-Ply laminates

[0/±φ]



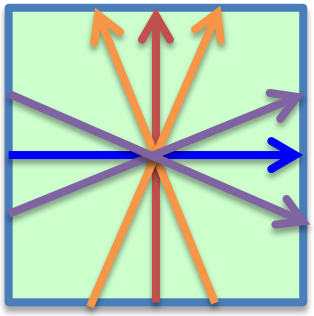
1-axis

[0/±φ/90]



2-axis

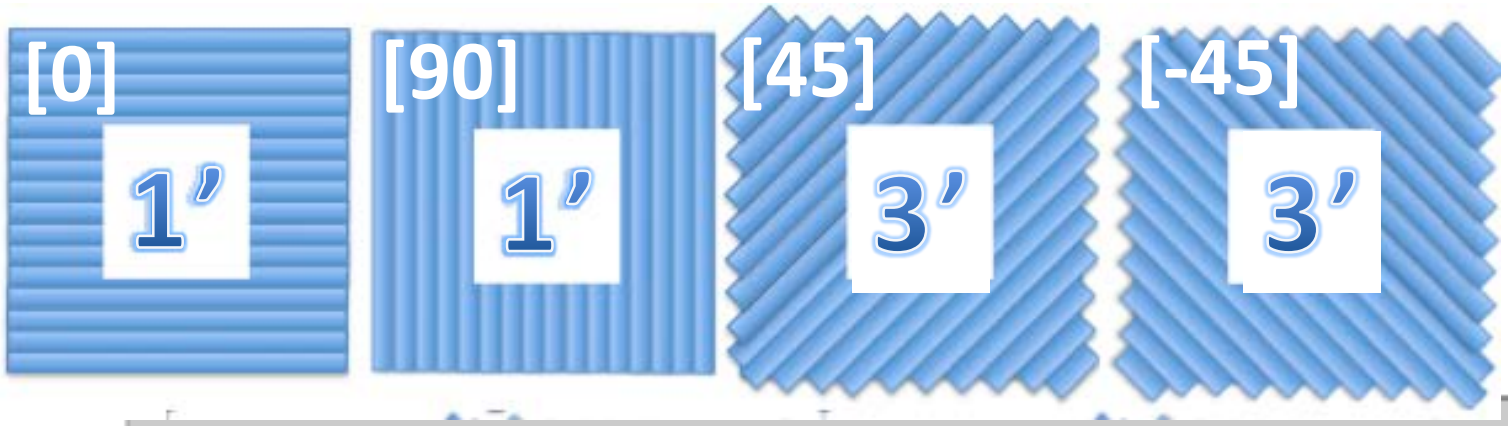
[0/±φ/±ψ/90]



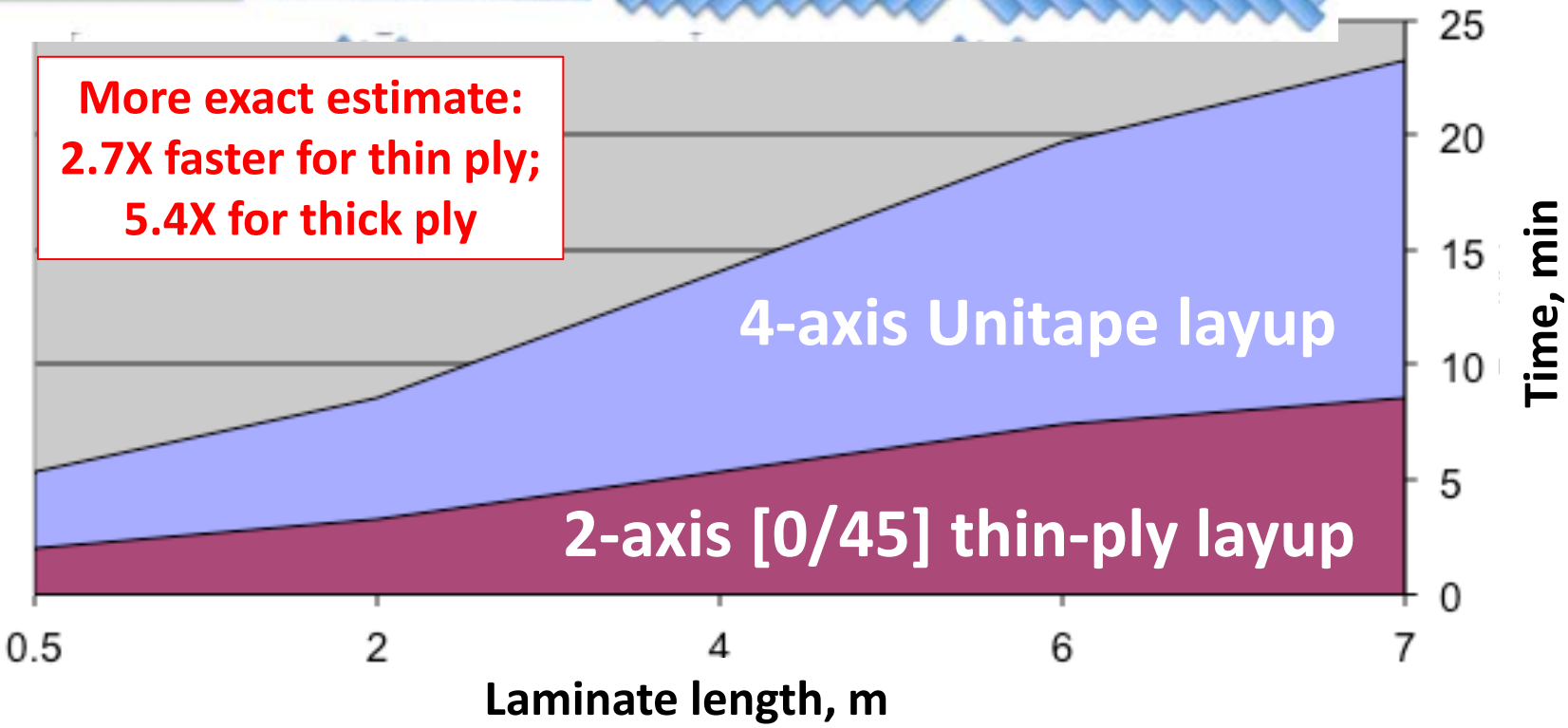
2-axis

Layup: 4-axis $[0]_{thk}$ vs 2-axis $[0/45]_{thn/thk}$

Jim Hecht of MAG



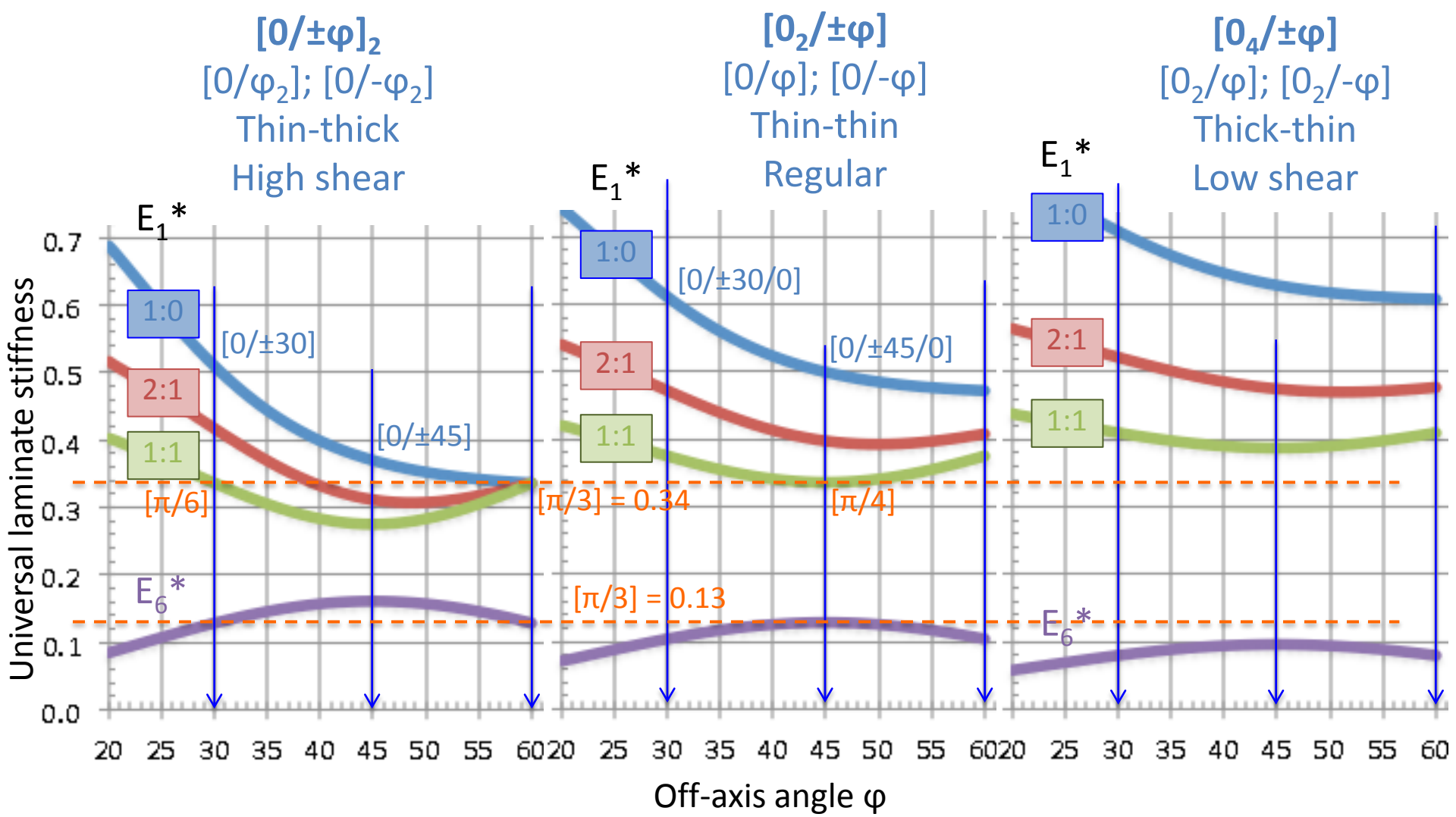
Layup time:
8' for thick [0]
4' for thin [0/45]
2' for thk [0/45]



High Speed Layup of Thick-thin C-Ply

Starting C-Ply	$[0/\varphi_2]$ - Thin-Thick (33/67/0) – 150 gsm	$[0/\varphi]$ - Thin-Thin (50/50/0) – 150 gsm	$[0_2/\varphi]$ - Thick-Thin (67/33/0) – 150 gsm
	High shear	Regular	Low shear
1-axis layup 1:0 5X	$[0/\pm\varphi]_2$ = $[\pi/3]_2$ for φ = 60 (33/67/0)	$[0_2/\pm\varphi]$ (50/50/0)	$[0_4/\pm\varphi]$ (67/33/0)
2-axis layup 2:1 3X	$[(0/\pm\varphi)_2/(\pm\psi/90)]_2$ $\Psi = 90 - \varphi$ (22/67/11)	$[(0_2/\pm\varphi)_2/\pm\psi_2/90_2]$ $\Psi = 90 - \varphi$ (33/50/17)	$[(0_4/\pm\varphi)_2/\pm\psi/90_4]$ $\Psi = 90 - \varphi$ (44/33/22)
2-axis layup 1:1 2X	$[0/\pm\varphi/\pm\psi/90]_2$ = $[\pi/6]_2$ for φ = 30 (17/66/17)	$[0_2/\pm\varphi/\pm\psi/90_2]$ = $[\pi/4]_2$ for φ = 45 (25/50/25)	$[0_4/\pm\varphi/\pm\psi/90_4]$ $\Psi = 90 - \varphi$ (33/33/33)

Universal Stiffness Components: C-Ply



Topics

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Black Aluminum Design: Bricky

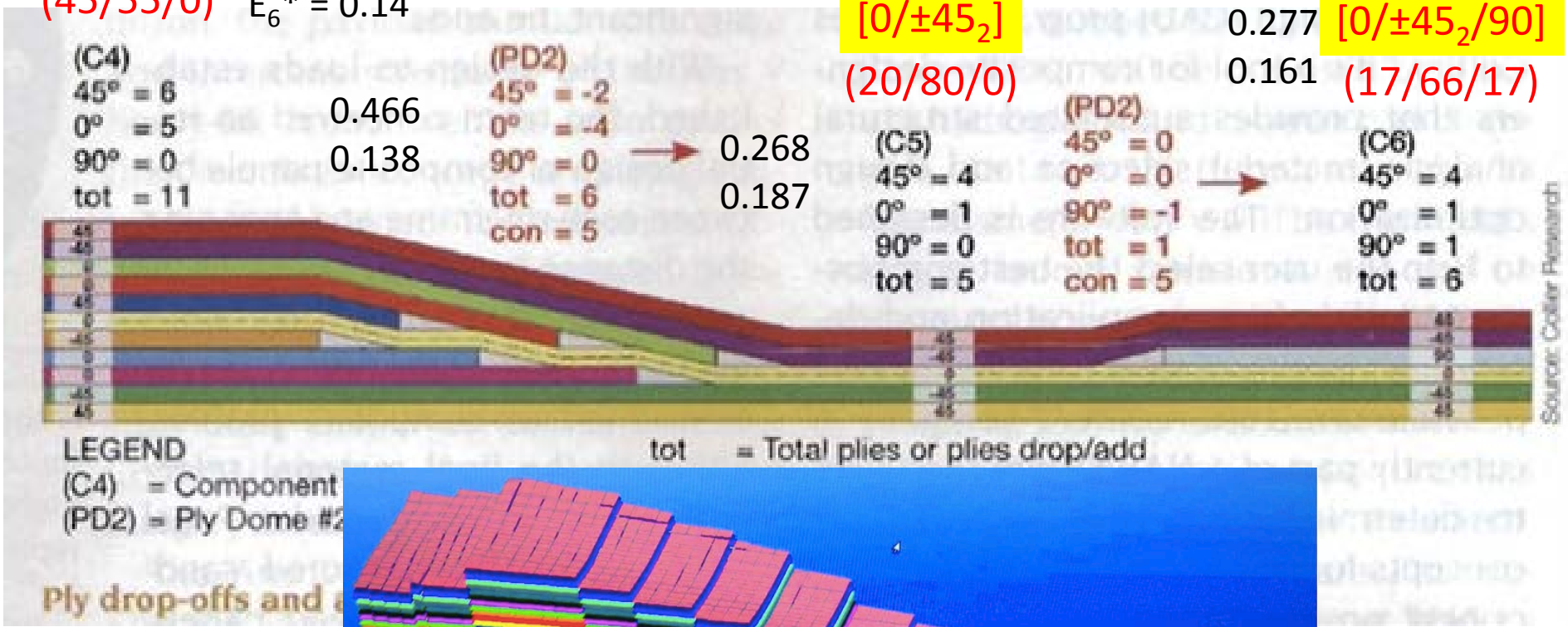
Cannot be optimized; manufacturing nightmare

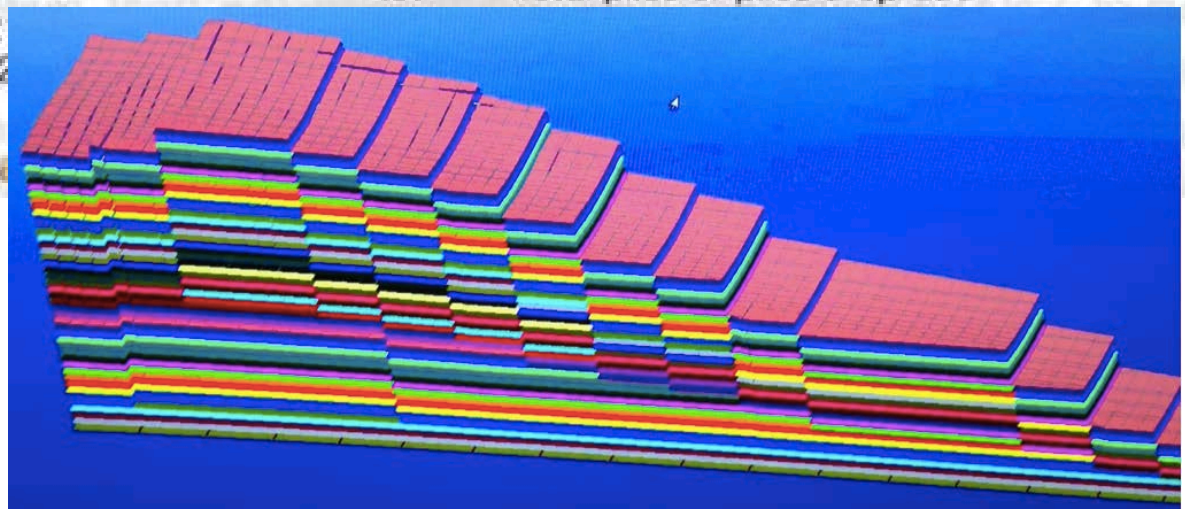
$[0_5/\pm45_3]$ (45/55/0)

$E_1^* = 0.45$
$E_6^* = 0.14$

$E_1^* = 0.22$
$E_6^* = 0.20$
$[0/\pm45_2]$ (20/80/0)

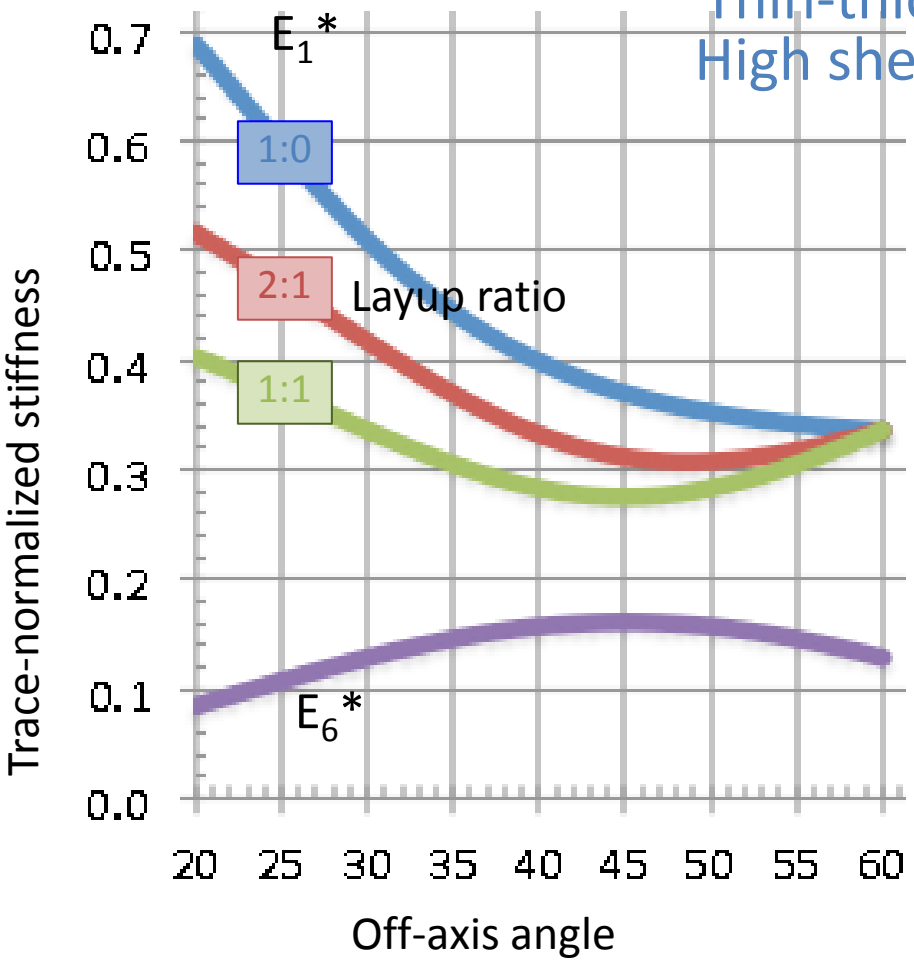
$E_1^* = 0.28$
$E_6^* = 0.16$
$[0/\pm45_2/90]$ (17/66/17)

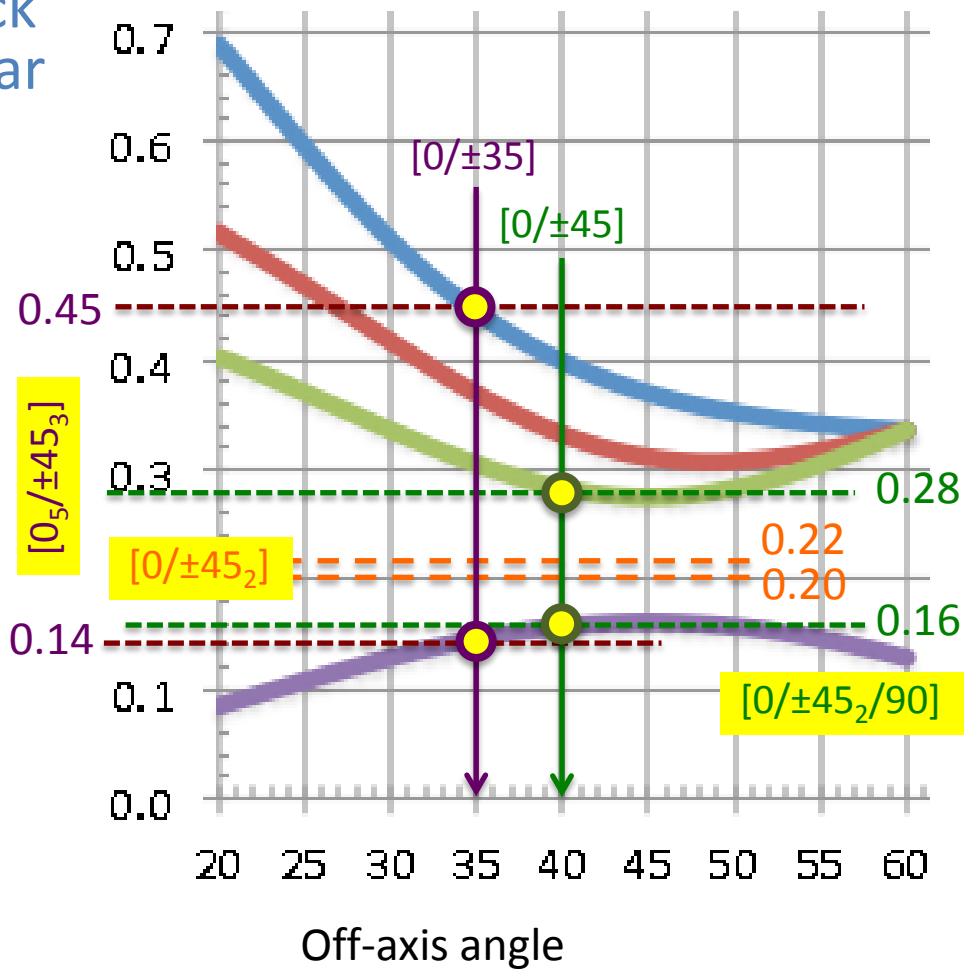




C-Ply: Black Aluminum Replacement

$[0/\pm\varphi]_2$
$[0/\varphi_2]$; $[0/-\varphi_2]$
Thin-thick
High shear





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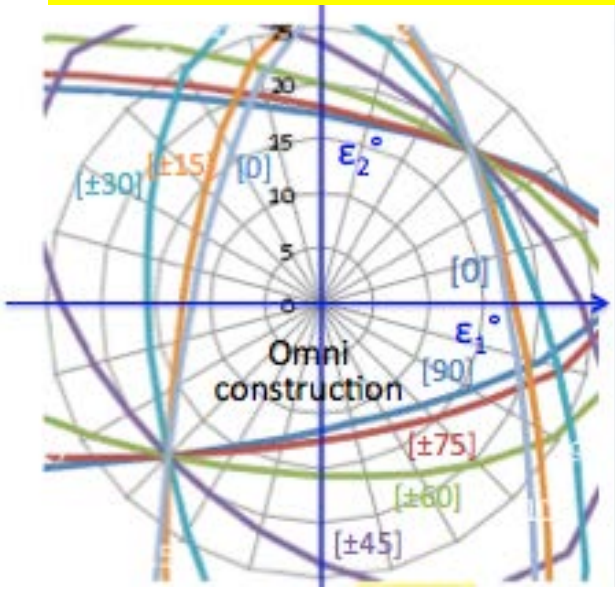
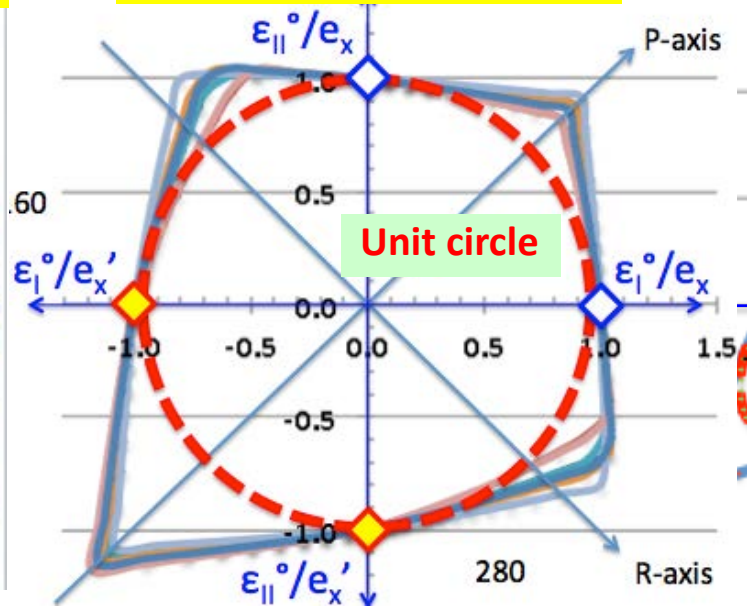
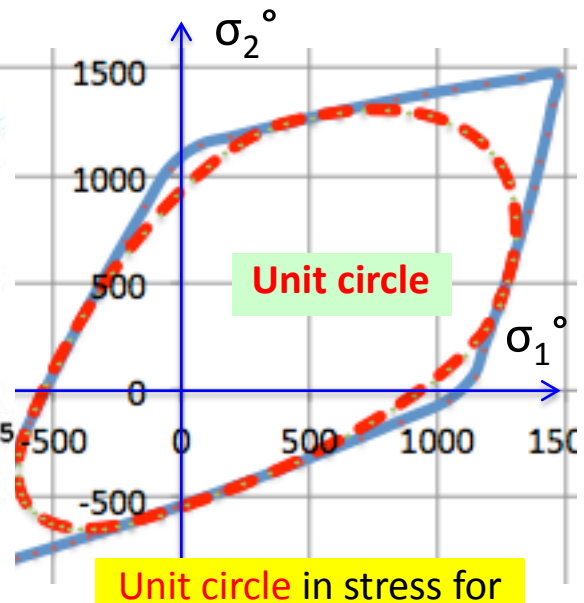
Evoluation: Unit Circle Failure Envelope

T700/2510; $E_m^* = 0.15$:
Fiber dominated omni envelope



One unit circle in strain for
all CFRP, all ply angles





Unit circle in stress for
IM7/977-3 [π/4] only

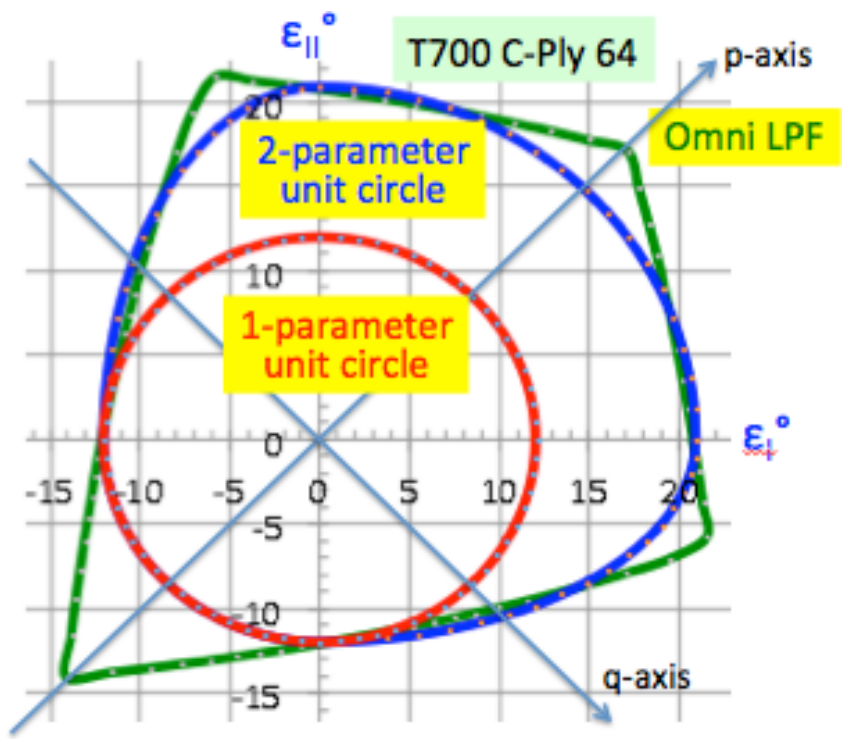
Safety Factor R; Failure Index k

R = Strength ratio: $[G_{ij}\epsilon_i\epsilon_j]R^2+[G_i\epsilon_i]R = aR^2+bR = 1$ **Scalable**

K = FEA index: $G_{ij}\epsilon_i\epsilon_j+G_i\epsilon_i = K^2$ **Go or no go only, not scalable**

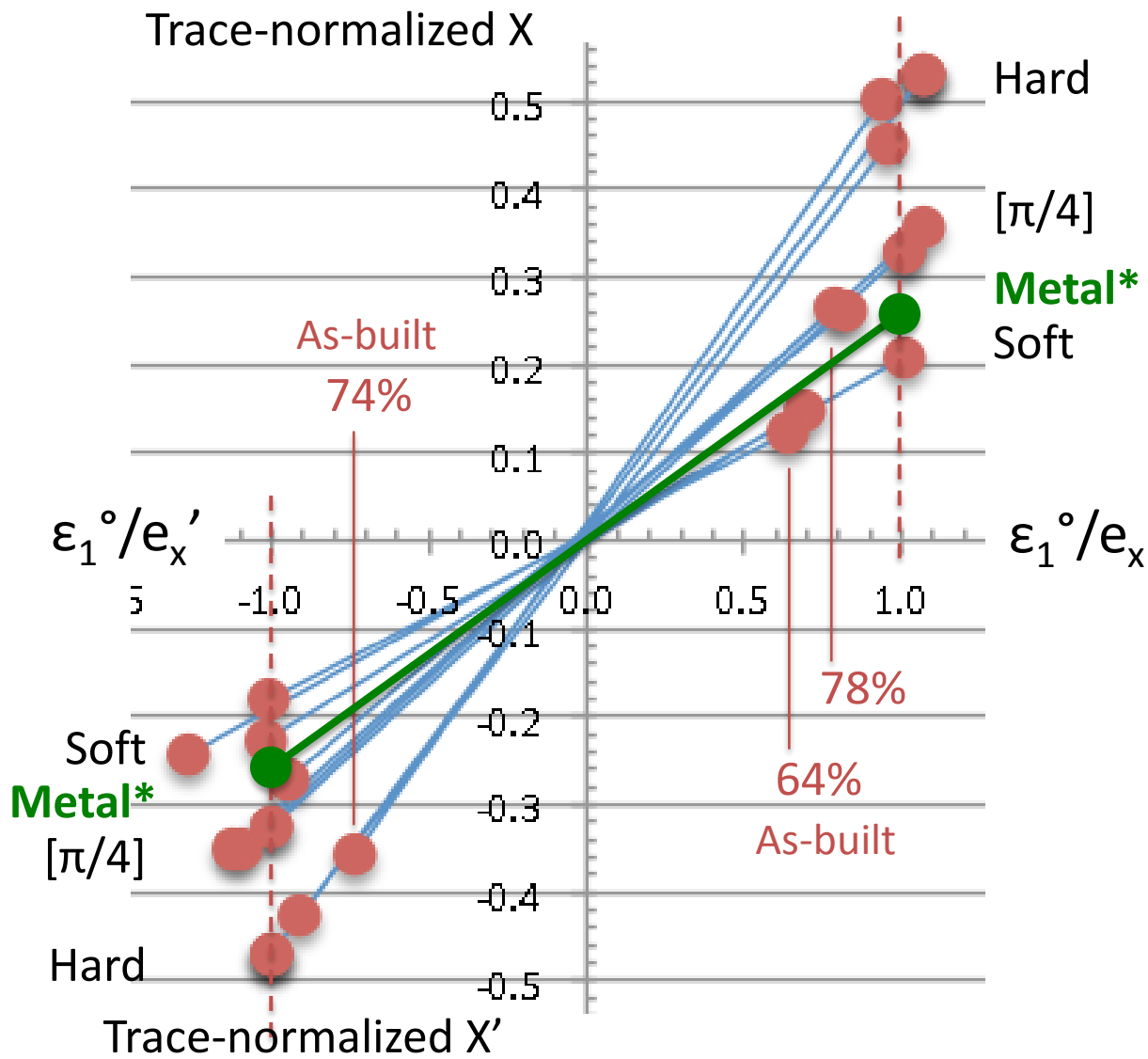
k = Failure index: $[G_{ij}\epsilon_i\epsilon_j]\left[\frac{1}{k}\right]^2+[G_i\epsilon_i]\left[\frac{1}{k}\right] = a\left[\frac{1}{k}\right]^2+b\left[\frac{1}{k}\right] = 1$

Scalable



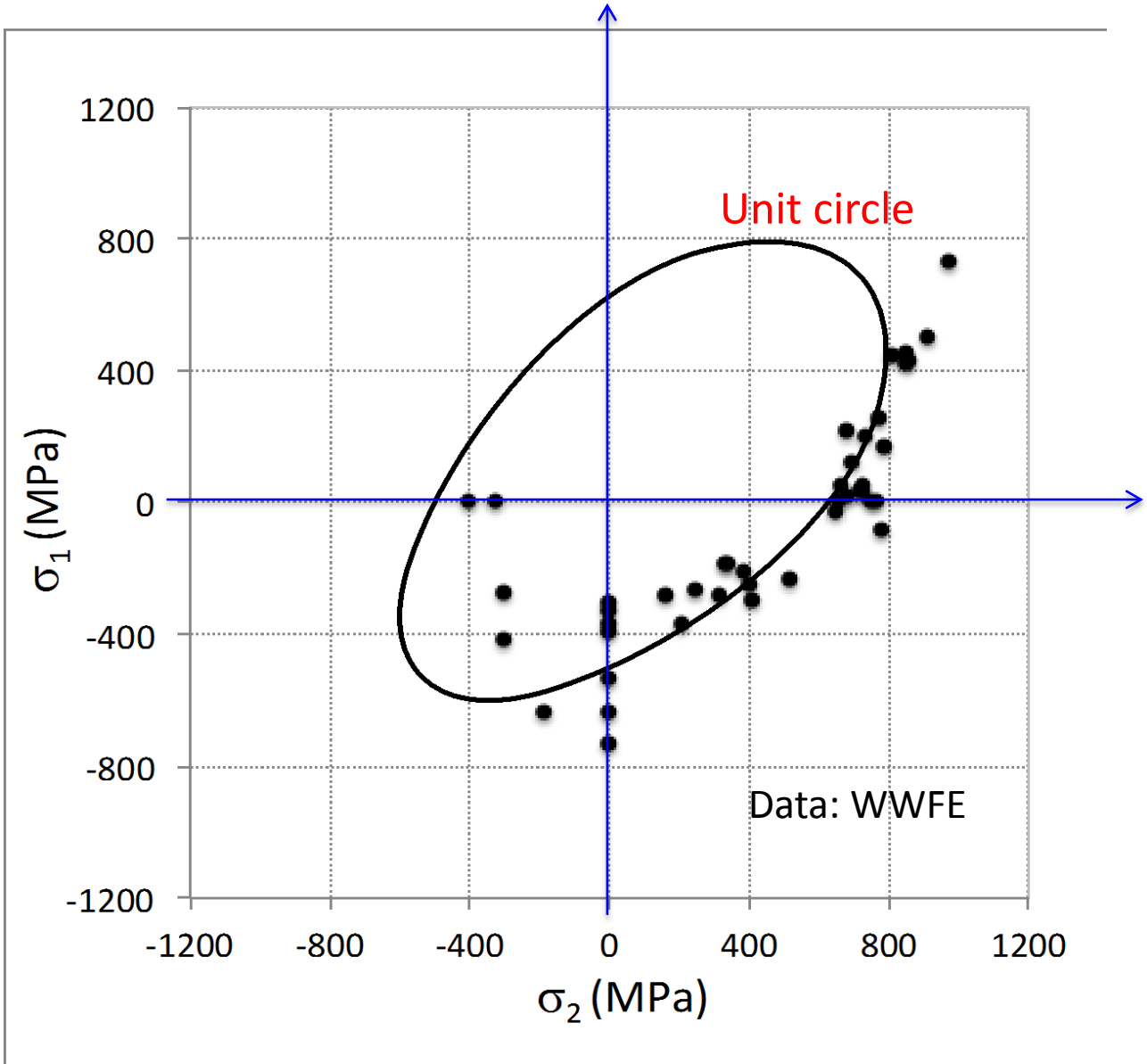
Universal Stress-Strain Curves for CFRP

	Laminate	Strain*	Stress*
T700/2510	[π/4]	0.79	0.27
		-1.10	-0.35
	Soft1	0.69	0.15
		-1.02	-0.23
	Hard1	1.07	0.53
		-1.00	-0.47
T650/epoxy	[π/4]	1.01	0.33
		-1.00	-0.33
	Hard1	1.07	0.53
		-1.00	-0.47
IM7/8552	[π/4]	1.01	0.33
		-1.00	-0.33
	Soft1	1.07	0.53
		-1.00	-0.47
	Hard1	0.96	0.45
		-0.91	-0.43
T4807/MR6(	[π/4]	0.82	0.26
		-0.95	-0.27
	Soft2	0.64	0.12
		-1.01	-0.18
	Hard2	0.94	0.50
		-0.74	-0.36
IM7/MTM45	[π/4]	1.07	0.36
		-1.13	-0.35
	Hard1	1.01	0.21
		-1.26	-0.24
	Median+	1.01	
	Median-	-1.00	



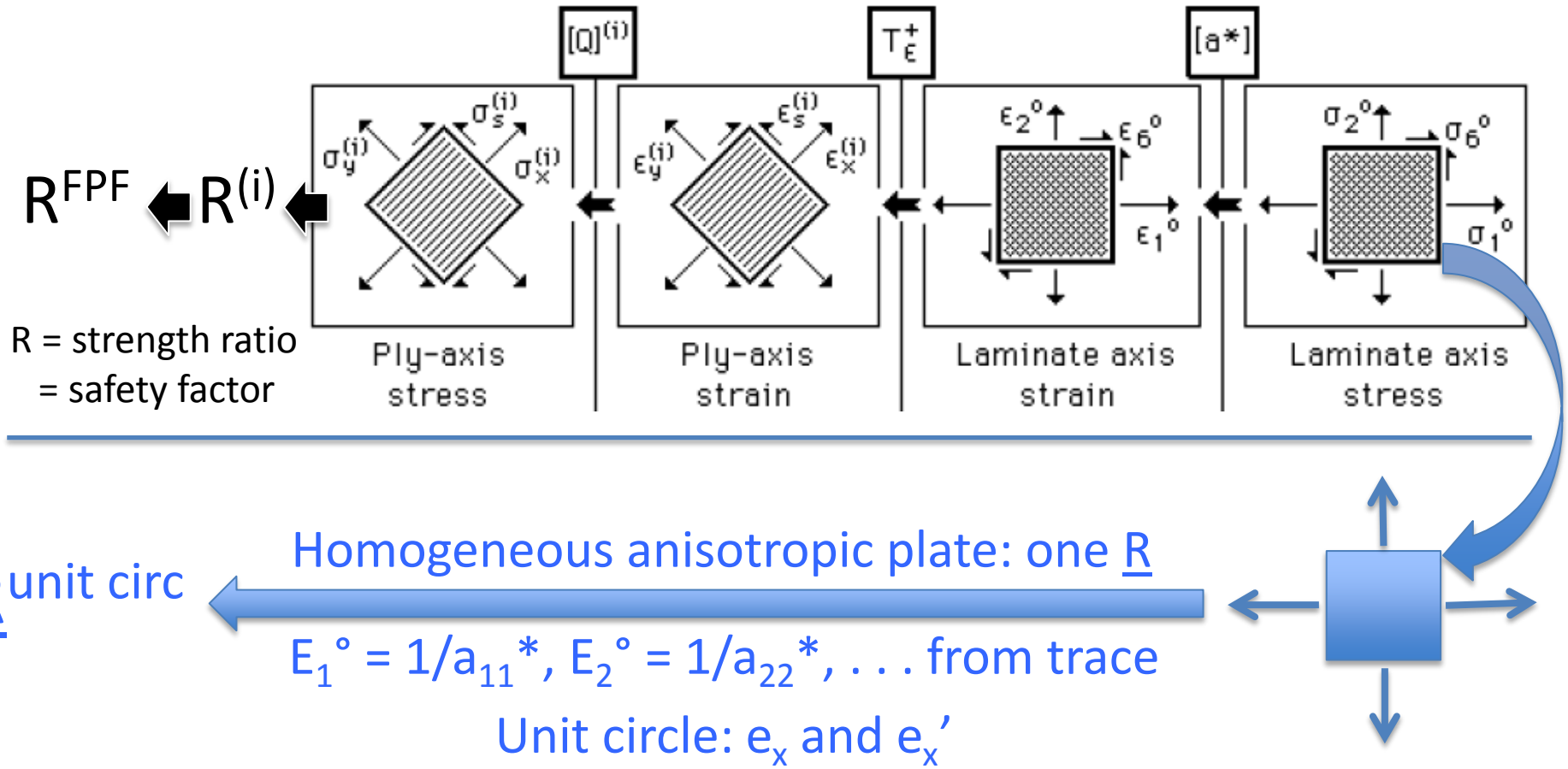
Metal*: Specific stiffness of Al, Ti and Fe (normalized by density)

Comparison Unit Circle vs WWFE Data



Ply-by-Ply vs Homogenized Plate

Ply-by-ply $R^{(i)}$ of a laminated anisotropic or orthotropic plate



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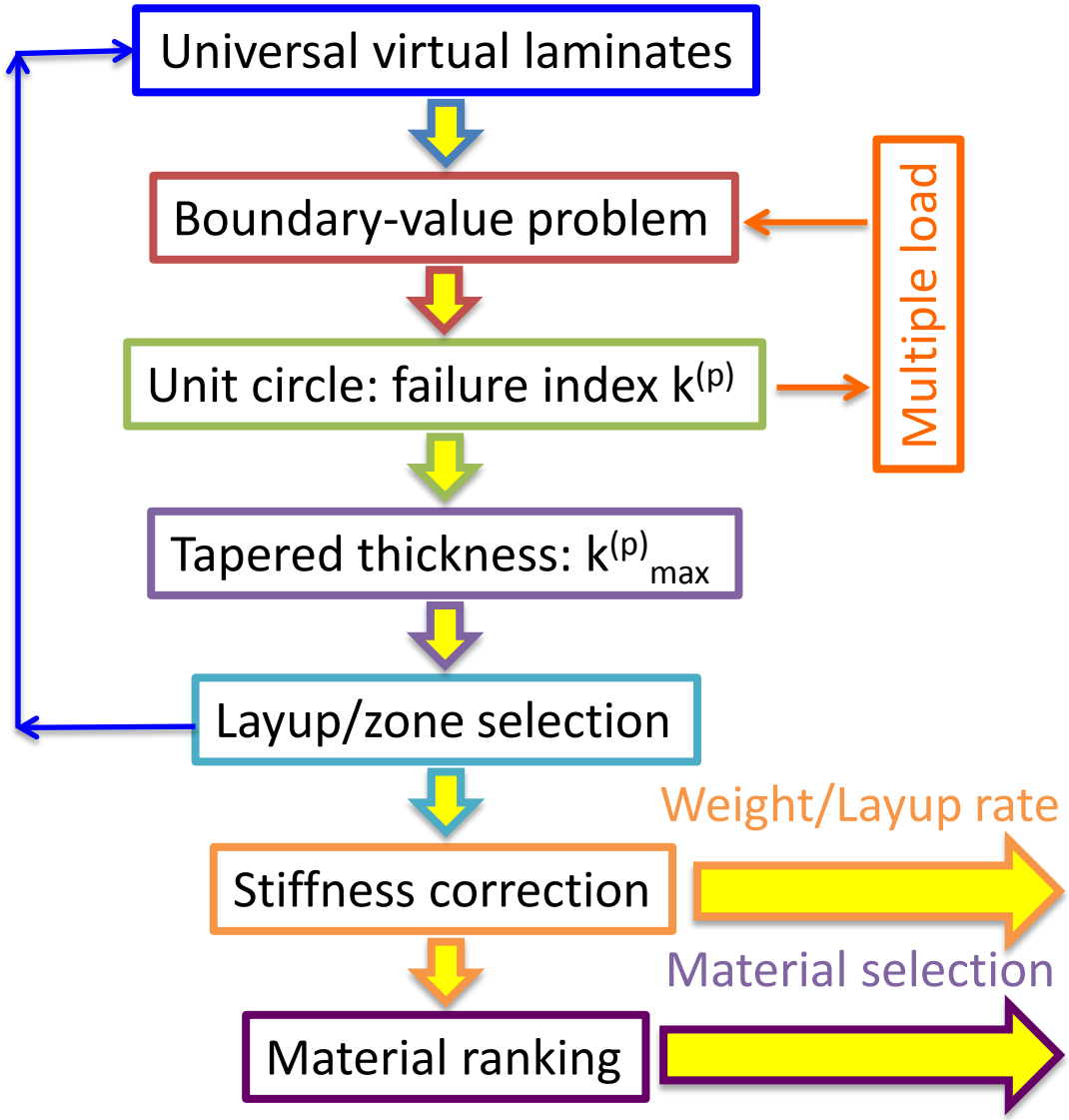
Direct Weight and Layup Rate Method

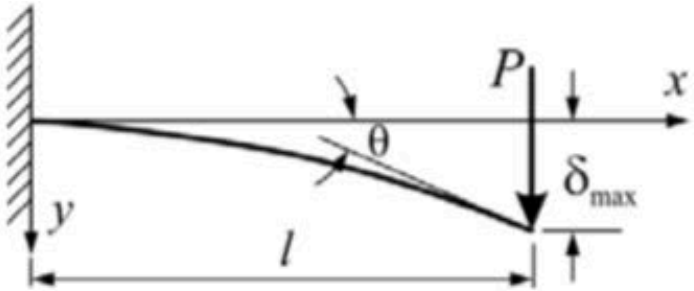


Essential ply properties

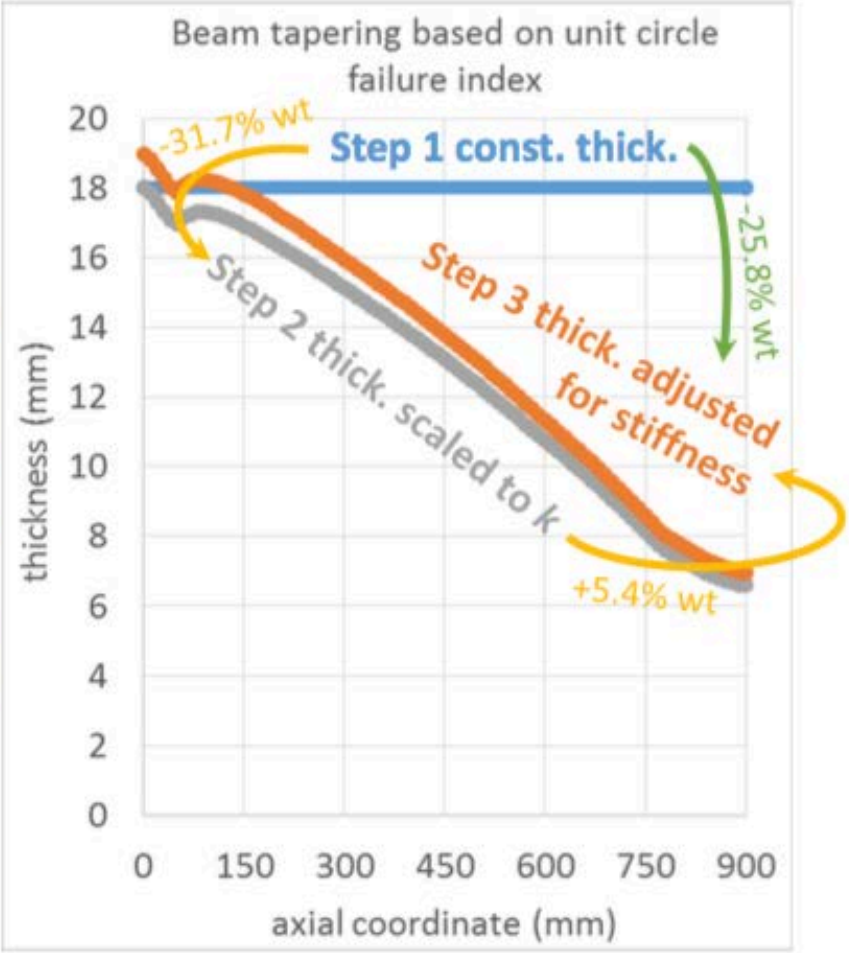
15 Ply material	Trace	Uniax tens e_x	Uniax comp e_x'
IM6/epoxy	232	17.2	7.6
IM7/977-3	218	17.0	8.4
T300/5208	206	8.4	8.4
IM7/MTM45	195	14.3	9.7
T800/Cytec	183	23.3	10.2
IM7/8552	180	15.7	10.7
T800S/3900	168	19.9	16.6
T300/F934	168	8.9	8.2
T700 C-Ply 64	163	20.9	14.1
AS4/H3501	162	10.5	10.5
T650/epoxy	160	15.8	11.9
T4708/MR60H	158	17.8	12.0
T700/2510	144	17.2	11.5
AS4/MTM45	143	14.7	11.0
T700 C-Ply 55	139	20.9	13.8

k-based Taper

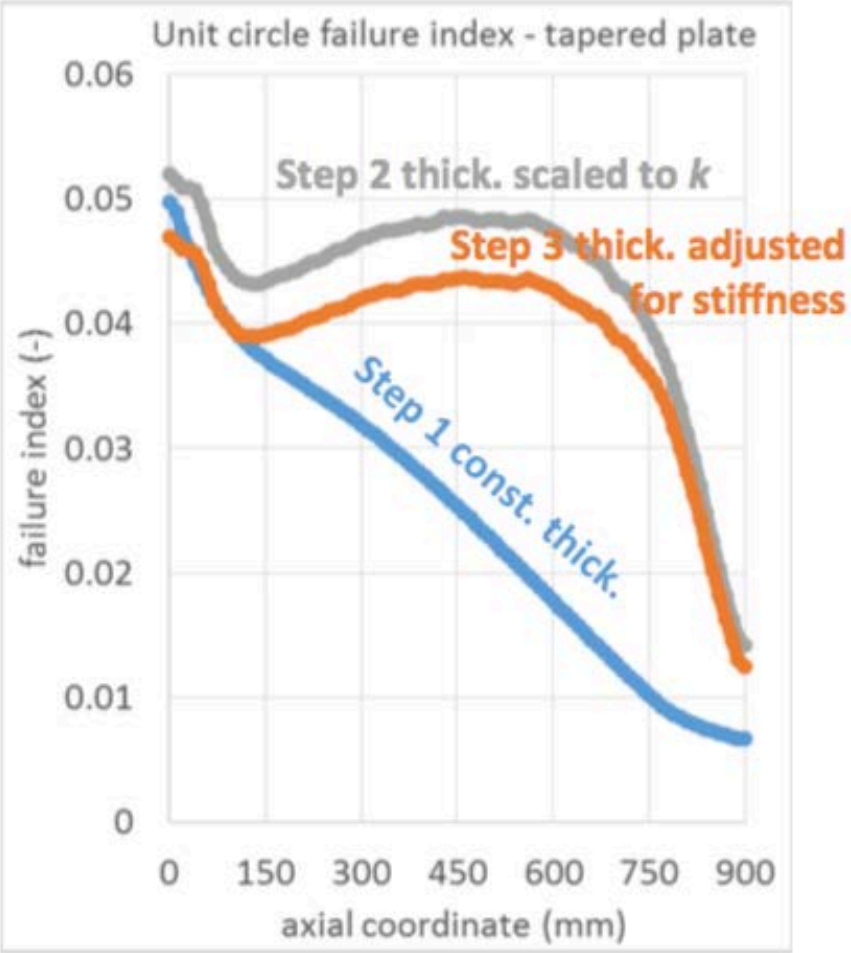
T700 C-Ply 64 [0/±45/0] laminate



Thickness variation



***k* variation**



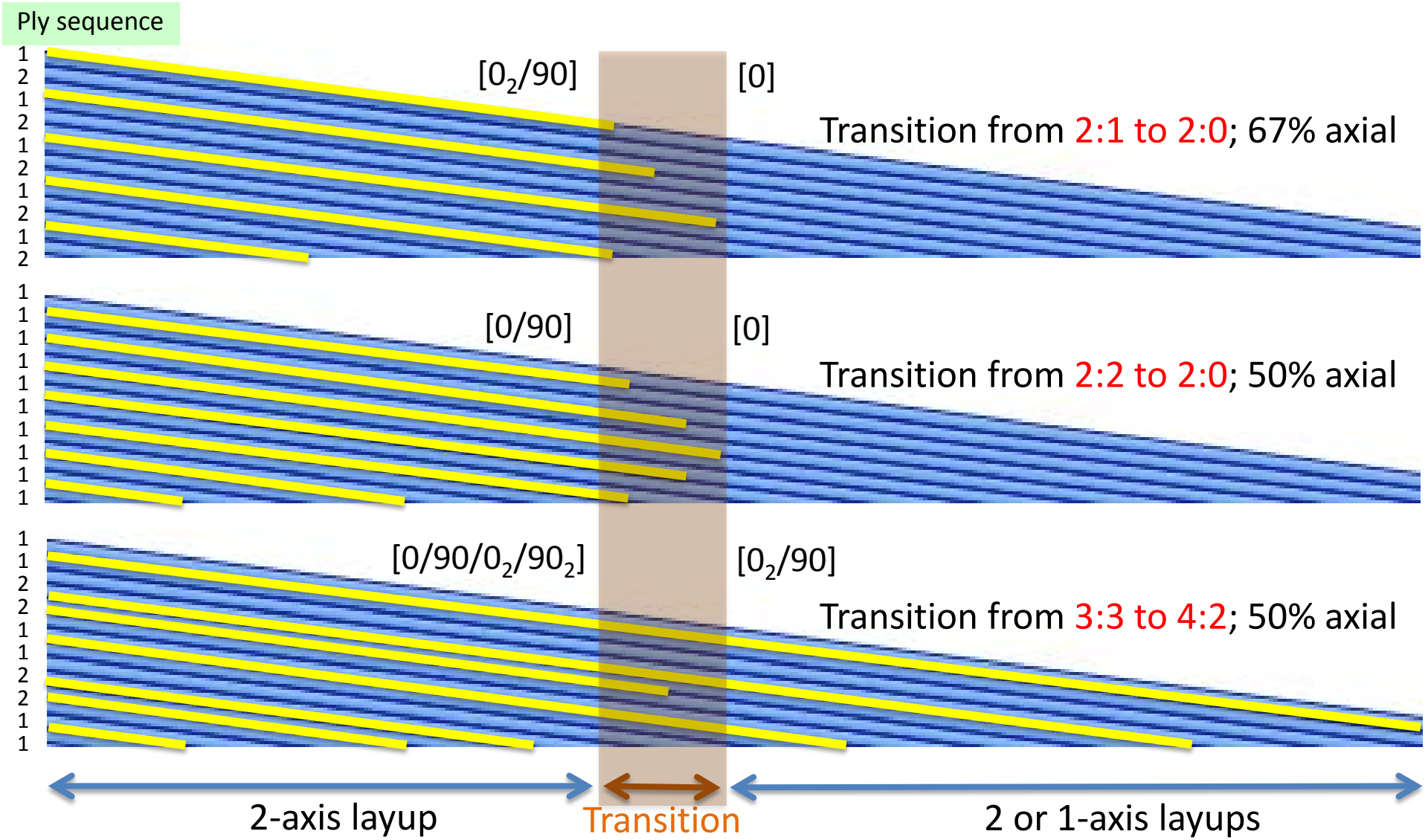
Examples of 1- and 2-axis Layup

1-axis 1:0 layup: Homogenized, single ply drop

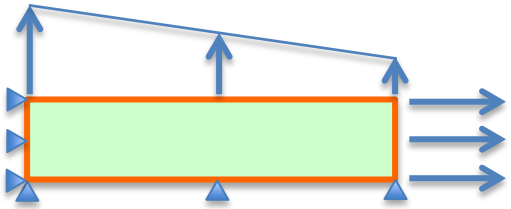
2-axis 2:1 layup: Homogenized, single ply drop

2-axis 1:1 layup: Homogenized, single ply drop

Examples of Transition between Layups



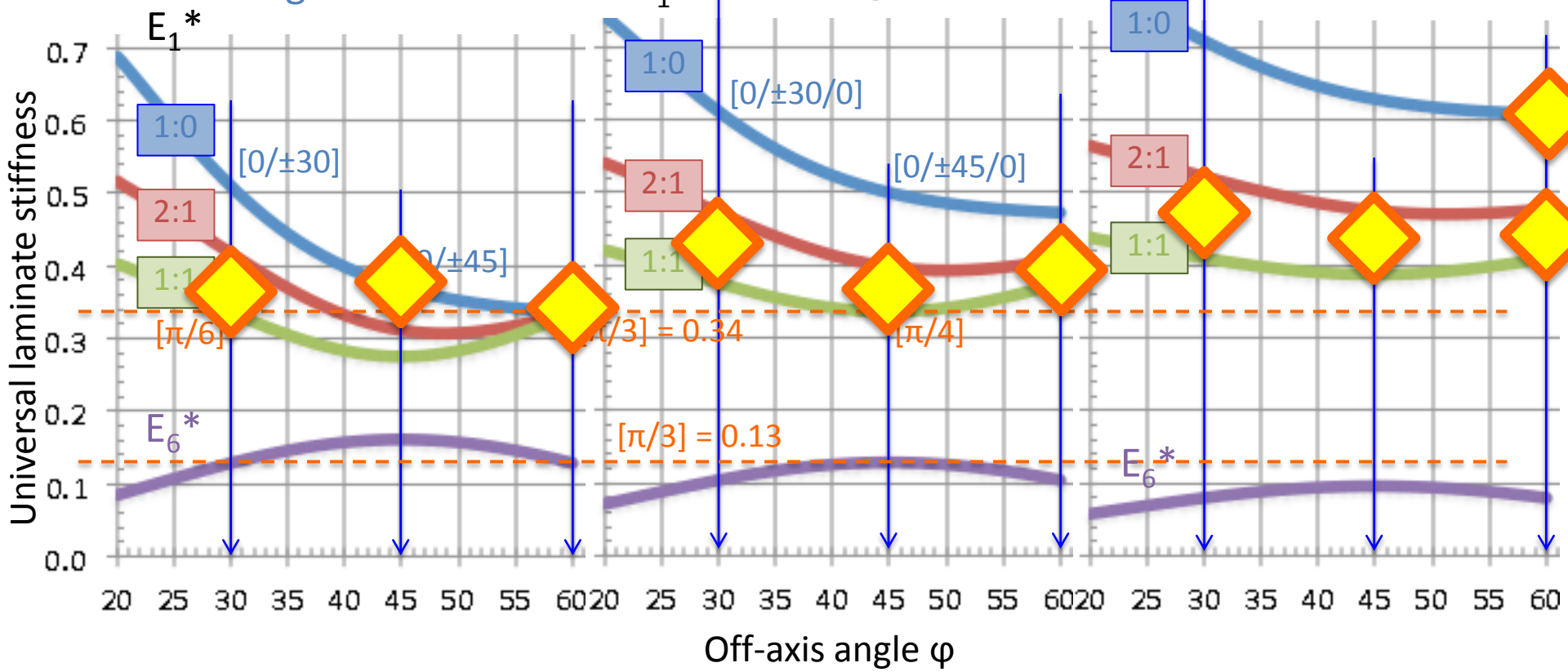
Cases Investigated



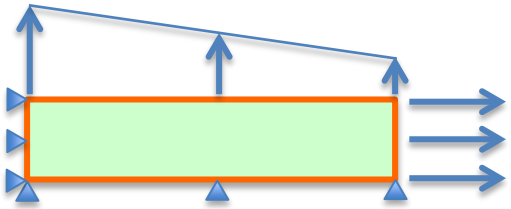
$[0/\pm\varphi]_2$
$[0/\varphi_2]$; $[0/-\varphi_2]$
Thin-thick
High shear

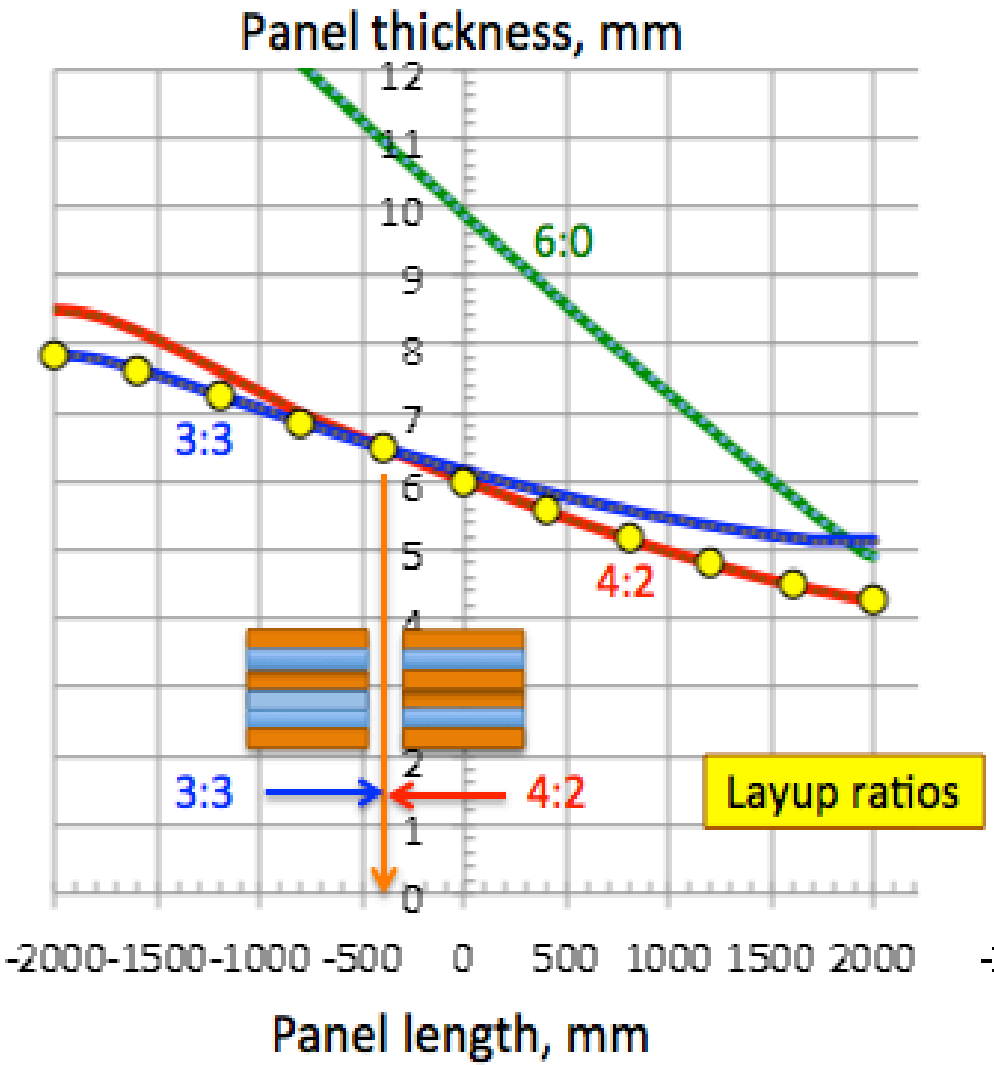
$[0_2/\pm\varphi]$
$[0/\varphi]$; $[0/-\varphi]$
Thin-thin
Regular

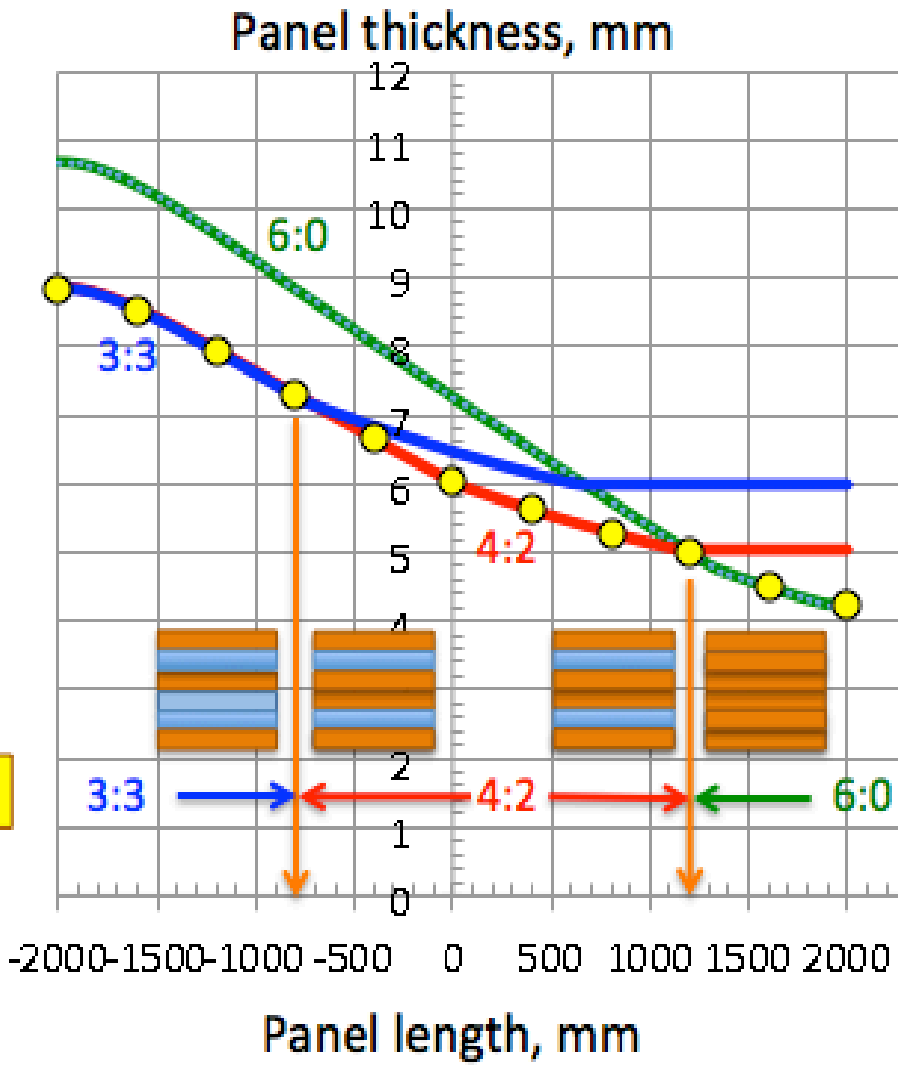
$[0_4/\pm\varphi]$
$[0_2/\varphi]$; $[0_2/-\varphi]$
Thick-thin
Low shear



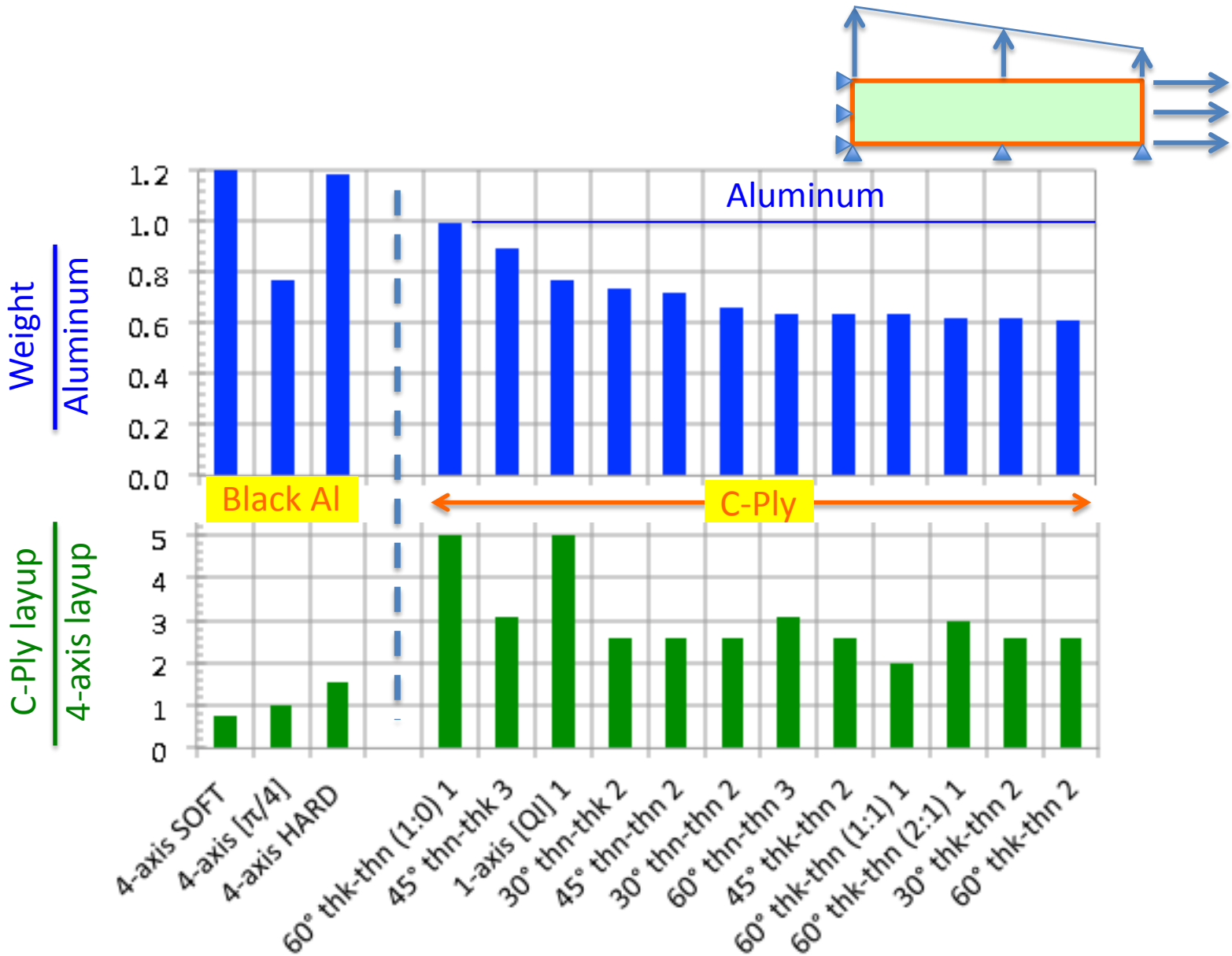
2- and 3-zone Layup







Weight Reduction and Layup Speed

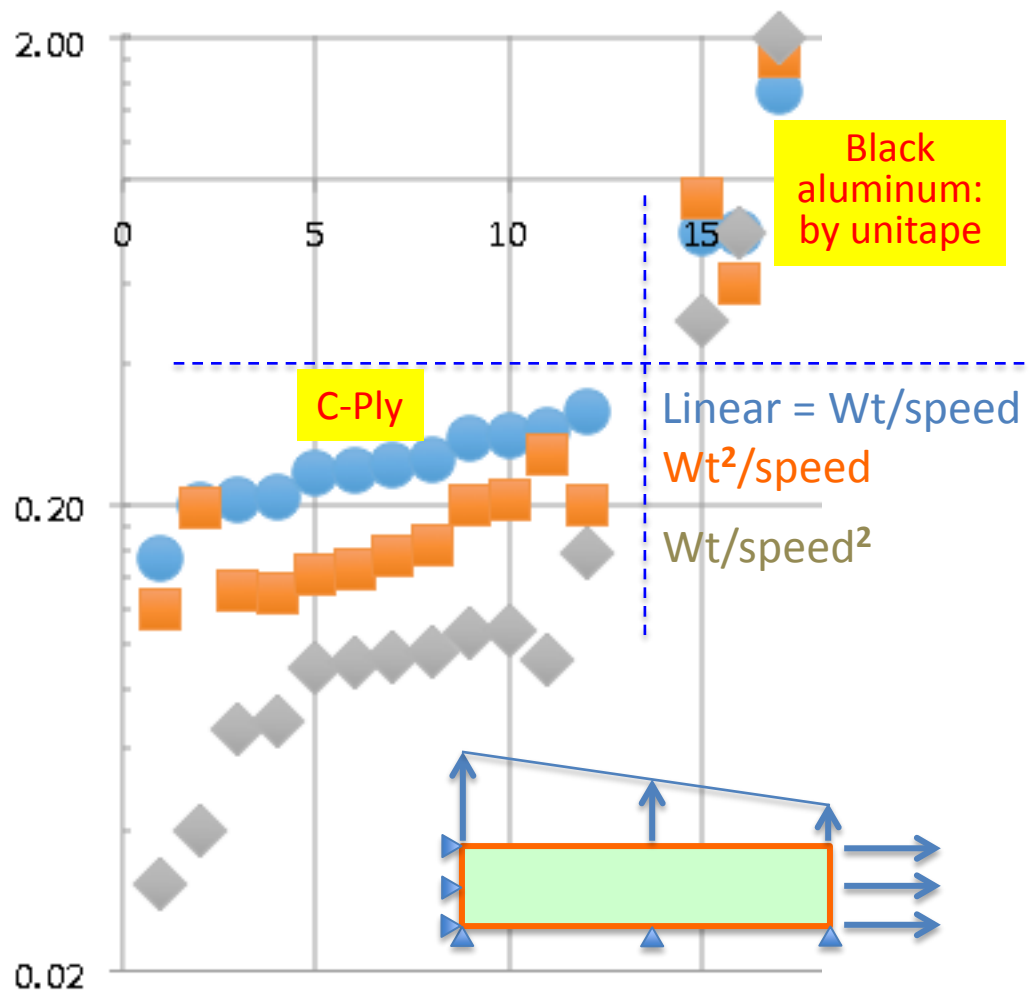


Cost Reduction: Weight/speed of C-Ply

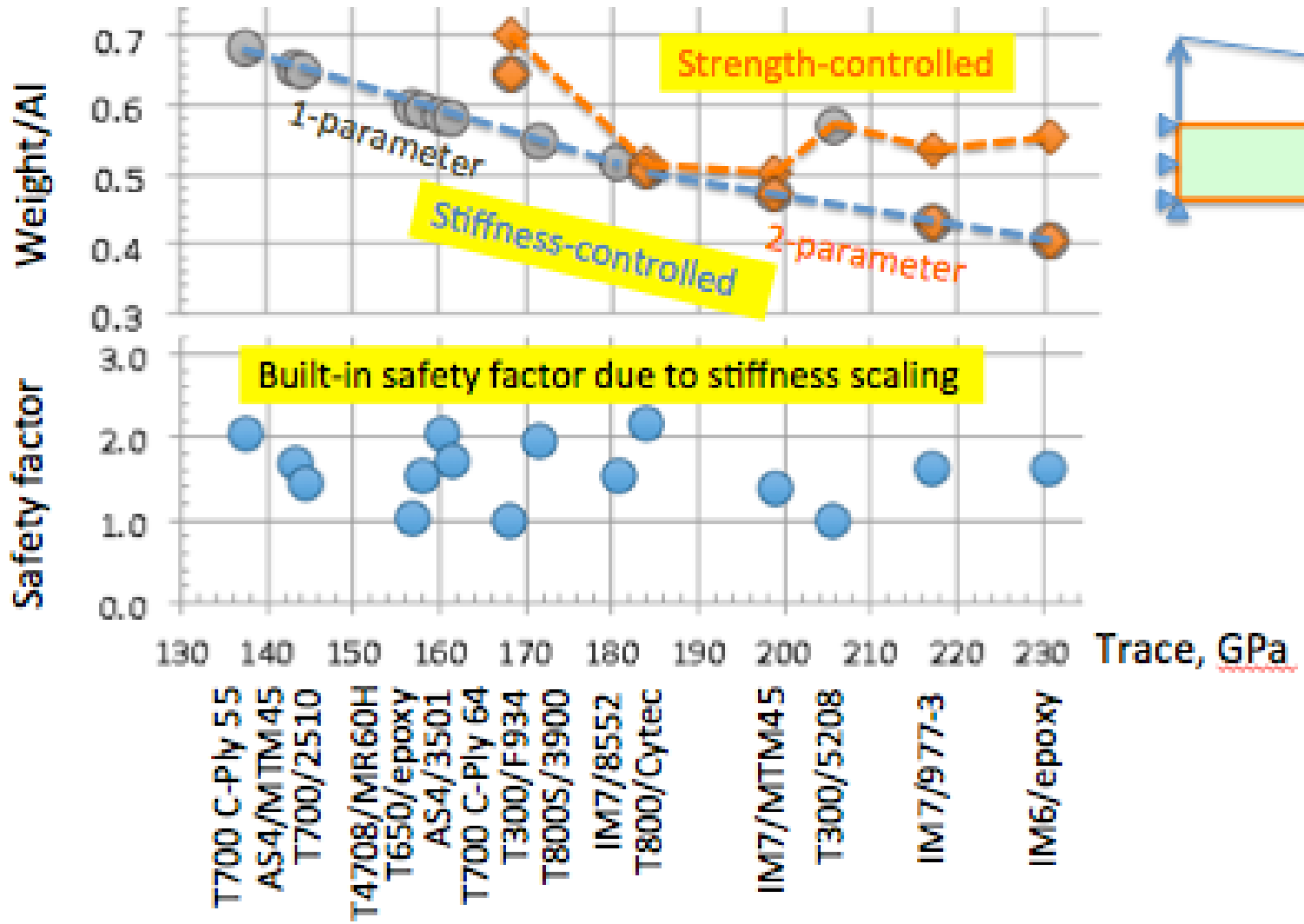
Ranking insensitive among values of exponents

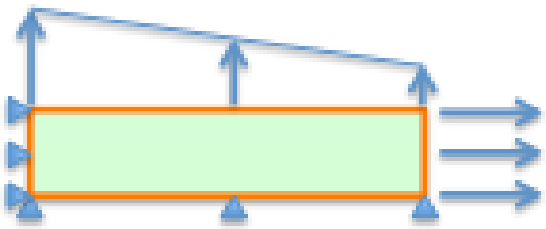
Distinct regions between black aluminum and C-Ply

Layup ranking	Linear	Wt^2	Speed^2
1-axis [π/3] 1	0.15	0.12	0.031
60° thk-thn (1:0) 1	0.20	0.20	0.040
60° thn-thn 3 2:2, 2:1 and	0.21	0.13	0.066
60° thk-thn (2:1) 1	0.21	0.13	0.069
60° thk-thn 2 2:2 and 2:1	0.23	0.14	0.090
30° thk-thn 2 2:2 and 2:1	0.24	0.15	0.091
45° thk-thn 2 2:2 and 2:1	0.24	0.15	0.094
30° thn-thn 2 2:2 and 2:1	0.25	0.16	0.097
45° thn-thn 2 2:2 and 2:1	0.28	0.20	0.107
30° thn-thk 2 2:2 and 2:1	0.28	0.20	0.108
45° thn-thk 3 2:2, 2:1 and	0.29	0.26	0.093
60° thk-thn (1:1) 1	0.32	0.20	0.158
Black aluminum			
4-axis HARD	0.77	0.91	0.499
4-axis [π/4]	0.77	0.60	0.771
4-axis SOFT	1.55	1.84	2.010



Material Selection for Weight Savings





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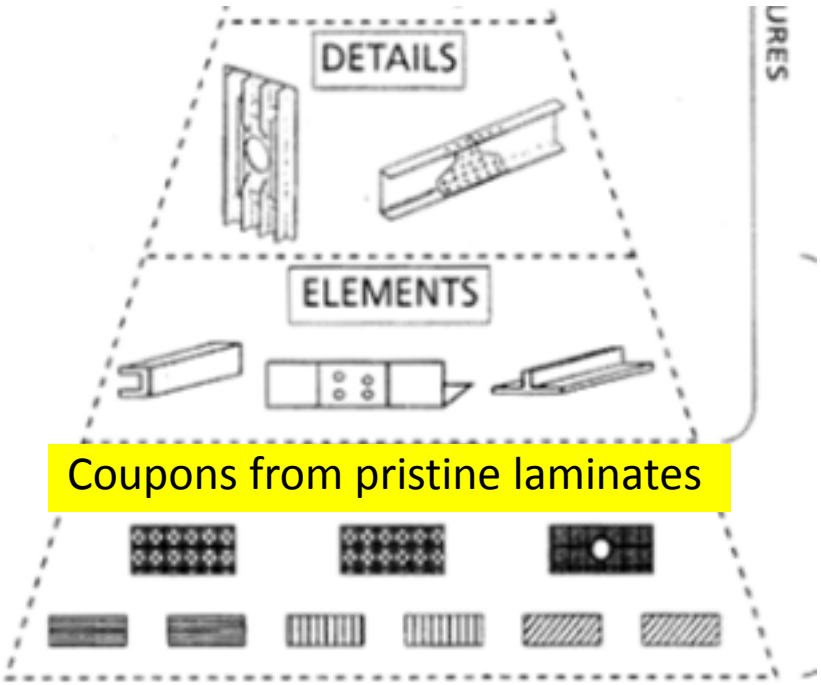
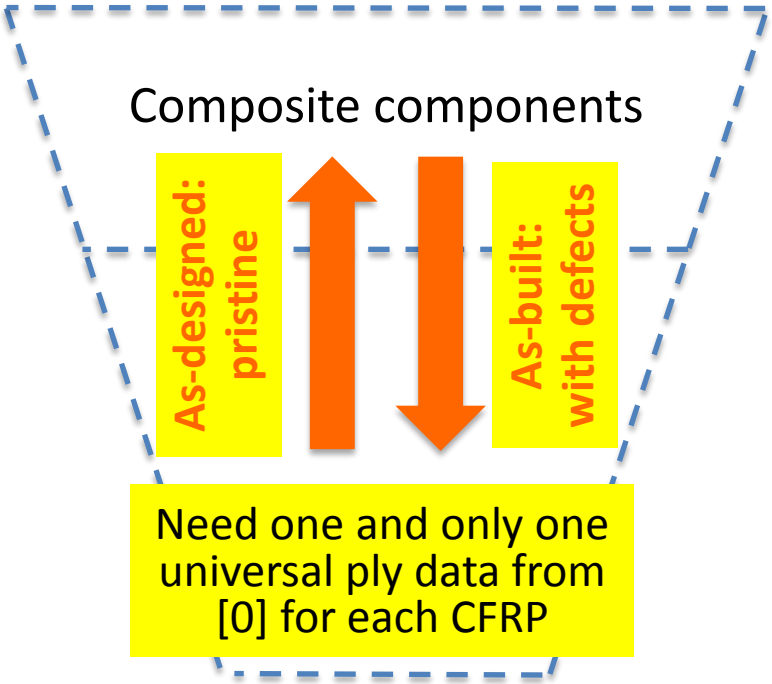
Traditional vs Trace-based Pyramids

Traditional building blocks



Trace-based: just like metals



Reduce coupons to one [0] and uniaxial tests
Unleash choices of laminates (not just 4), and
Include processing defects in as-built coupons

Black Aluminum vs Trace: 15 CFRP

E_x = 0.88 Trace

Black aluminum vs Trace

Ply stiffness

E_y

ν_x

E_s

Ply strength

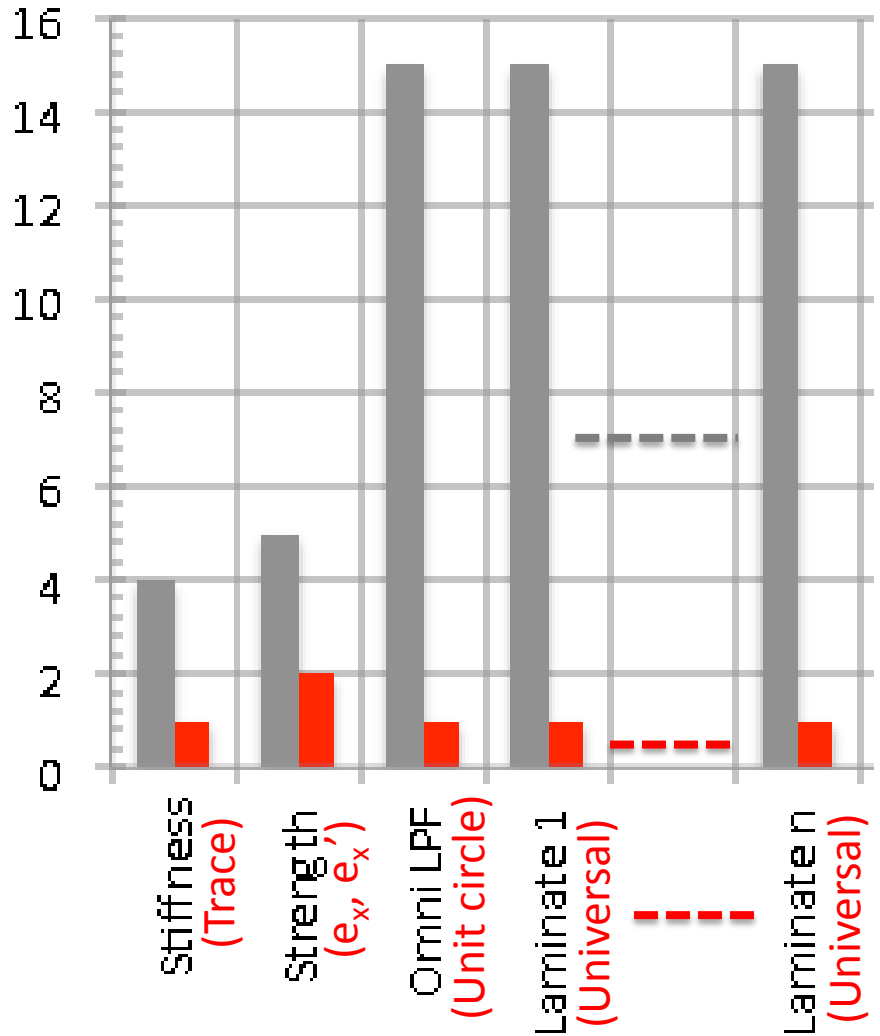
X

X'

Y

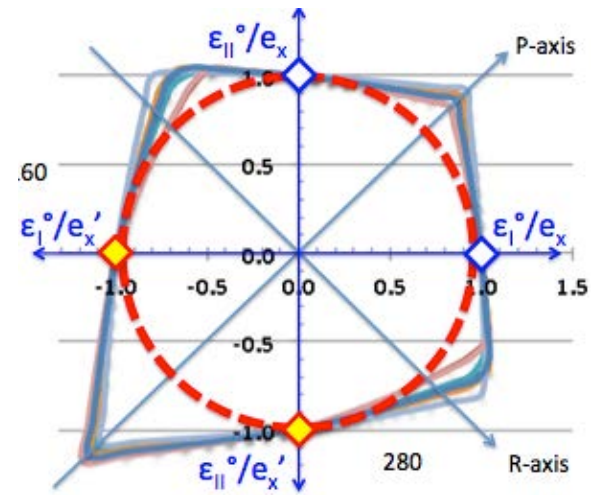
Y'

S

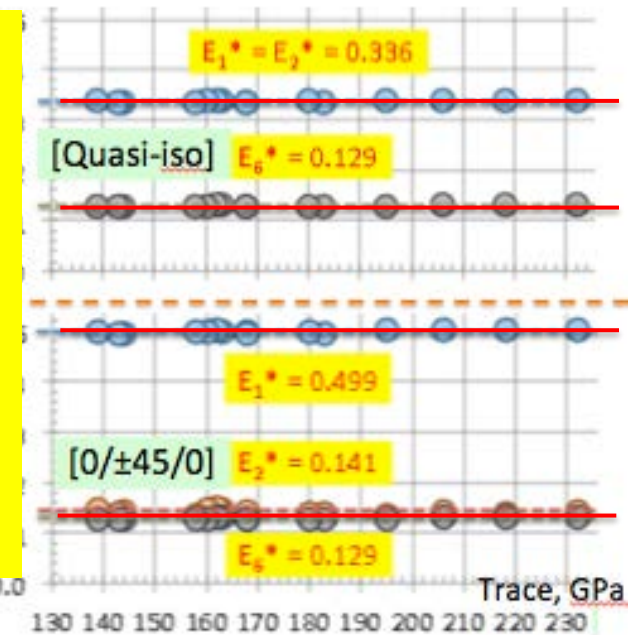


So much simpler: 4:1 5:2 15:1 15:1 ---- 15:1

Unit circle



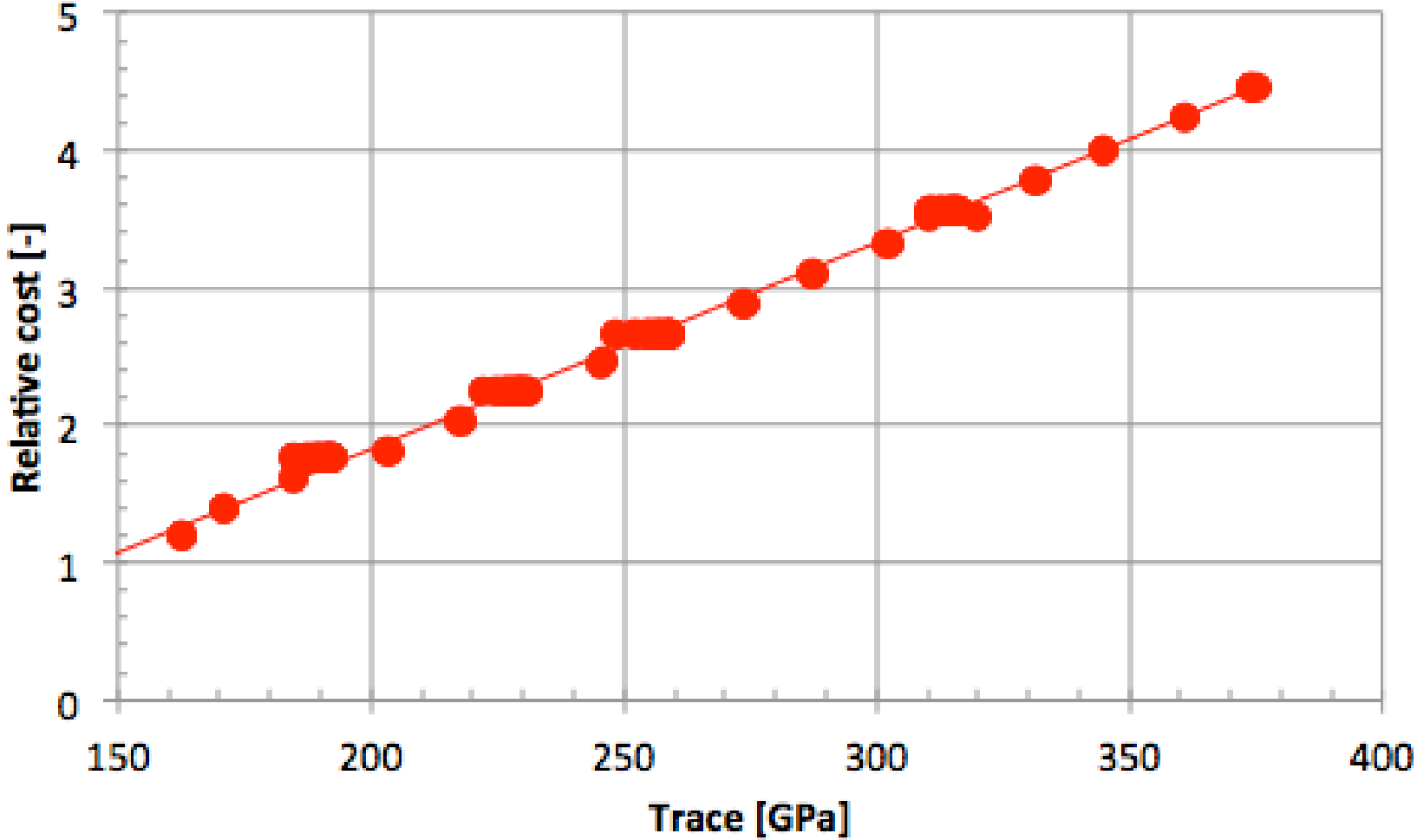
Universal Laminates



Relative Cost of Hybrid Laminates

Linear correlation with trace, but not with E_x or others

$$\text{Trace}^{\text{Hybrid}} = v_1\text{Trace}_1 + v_2\text{Trace}_2 + \ldots$$



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February 1-5, 2016; noon to 4 PM PST; 20 hours + homework

US$1,200 including hardcover and e-books, composites app

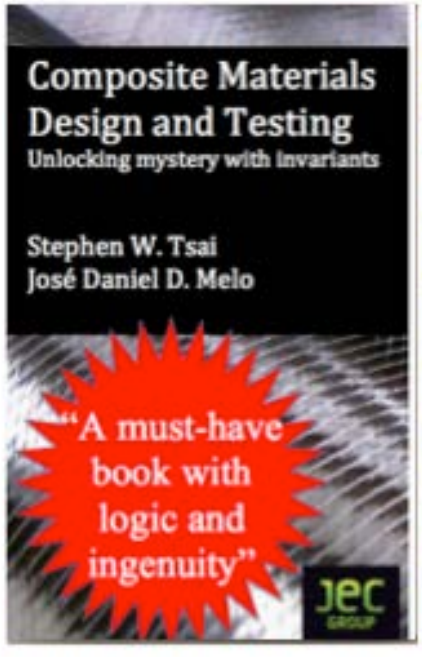
MS Excel-based MicMac's and other practical design tools

All sessions recorded/downloadable for individual viewing

Widely recognized as the best online training; no travel

Optional official transcript of 3 credit CE hours for extra fee

Must-learn trace that has revolutionized composites testing



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