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The effect of acceleration on turbulent entrainment

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Abstract
A new class of self-similar turbulent flows is proposed, which exhibits dramatically reduced entrainment rates. Under strong acceleration, the rotation period of the large-scale vortices is forced to decrease linearly in time. In ordinary unforced turbulence, the rotation period always increases linearly with time, at least in the mean. However, by imposing an exponential acceleration on the flow, the vortex rotation period is forced to become the e-folding timescale of the acceleration. If the e-folding timescale itself decreases linearly in time, the forcing is 'super-exponential', characterized by an acceleration parameter α. Based on dimensional and heuristic arguments, a model suggests that the dissipation rate is an exponential function of α and the dimensions of the conserved quantity of the flow. Acceleration decreases the dissipation and entrainment rates in all canonical laboratory flows except for Rayleigh–Taylor. Experiments of exponential jets and super-exponential transverse jets are in accord with the model. As noted by Johari, acceleration is the only known means of affecting the entrainment rate of the far-field jet. Numerical simulations of Rayleigh–Taylor flow by Cook and Greenough are also consistent. In the limit of large acceleration, vortices do not move far before their rotation period changes substantially. In this sense, extreme acceleration corresponds to stationary vortices.

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1. Introduction

It has long been known that pressure gradients strongly affect the boundary layer. A negative pressure gradient can completely relaminarize a turbulent boundary layer (Narasimha and Sreenivasan 1973, 1979, Narasimha 1983). From the nonlinear momentum equation, a negative pressure gradient makes the velocity profile more full. The no-slip boundary condition implies that the mean spanwise vorticity profile becomes narrower and more sharply peaked at the wall. So the mean spanwise vorticity must increase. Since the average vortex rotation period is proportional to the inverse of the mean vorticity, the average vortex rotation period is reduced.

In the turbulent wake, a negative pressure gradient also affects the flow, but in a different way from that of the boundary layer. The mean spanwise vorticity in the wake is reduced in a negative pressure gradient. Narasimha and Prabhu (1972), Prabhu and Narasimha (1972), and Liu *et al* (1999) found non-self-similar behavior with pressure gradients. Keffer (1965, 1967) and Elliott and Townsend (1981) could not find self-similar flow when a plane wake was subjected to a constant strain rate of varying orientations. Rogers (2002) distorted plane wakes at constant rate in direct numerical simulations. He also found that the flows were not self-similar. This suggests that any self-similar flow must correspond to a different type of forcing. As discussed below, a new theory suggests that acceleration effects can be understood in terms of the temporal evolution of the vortex rotation period. In this way, a new class of self-similar flows is proposed.

Since vorticity has dimension of inverse time, one might speculate that time is the natural language of a vortex. If so, how do turbulent vortices respond when a single timescale is imposed on them? Oster and Wygnanski (1982) discovered that the imposition of a sinusoidal perturbation of fixed period had a dramatic effect on the entrainment rate of the free shear layer. It is natural to wonder about an exponential forcing of constant e-folding timescale.

A simple argument suggested that the dimensionless entrainment rate into a jet with an exponentially increasing nozzle speed would be reduced by the acceleration (Breidenthal 1986). Entrainment at the rear of a vortex might be inhibited if the neighboring vortex was increased in strength. Kato *et al* (1987) measured the visible flame length of a chemically reacting jet with exponential forcing. They found that the flame length of the exponential jet was about 20–25% longer than the classical jet. Since the mixing is entrainment-limited at large Reynolds number, their result indicated a reduction in the normalized entrainment rate.

More careful and detailed measurements by Zhang and Johari (1996) revealed the internal structure of the exponential jet. Their images of the concentration field demonstrated that the acceleration must be appreciable for any dynamic effect on the normalized entrainment. If the imposed velocity change during one vortex rotation is too small, then the entrainment rate into that vortex is not affected by the acceleration. So a threshold must exist for any dynamic effect. Zhang and Johari also demonstrated that the vortex rotation period was equal to the e-folding time of the exponential nozzle flow, as expected. In other words, each large-scale vortex in the far field rotated at the same rate, no matter how far from the nozzle. The exponential forcing *imposed* its e-folding timescale on the vortex rotation period. This key concept will be exploited below.

As noted by Johari, acceleration is the only known means of altering the entrainment into the far-field turbulent jet. Nothing else done at the nozzle has a lasting effect on the jet. Acceleration is a powerful tool for controlling turbulence.

2. Theory

Recently, a simple theory has been developed in an attempt to account for acceleration in turbulent flows in general (Breidenthal 2003). The approach assumes that the most important property of a vortex is its rotation period. Acceleration affects turbulent entrainment depending on how the vortex rotation period $\tau_v(t)$ changes with time t.

2.1. Self-similarity

In order for the flow to be strictly self-similar, the rotation period should change by a constant factor at each rotation. This implies that $\tau_v(t)$ must be linear, at least in the mean. For ordinary, unaccelerated turbulence, $\tau_v(t)$ increases in time as the rotation period lengthens in the natural aging process. On the other hand, in the exponential jet, $\tau_v(t)$ is a constant, equal to the e-folding timescale of the exponential forcing.

Extrapolating from the unforced to the exponential jet, one might expect the dimensionless entrainment rate to decline even further if the e-folding timescale would decline in time. Suppose that the e-folding timescale is selected to decline linearly with t. If the forcing is successful, the rotation period of the vortices is obliged to follow the e-folding timescale of the forcing. The linear nature of the e-folding timescale implies that the vortices would be self-similar.

Figure 1 illustrates the evolution of the vortex rotation period for these three cases, where $\tau_v(t)$ either increases, remains constant, or decreases with t. According to our

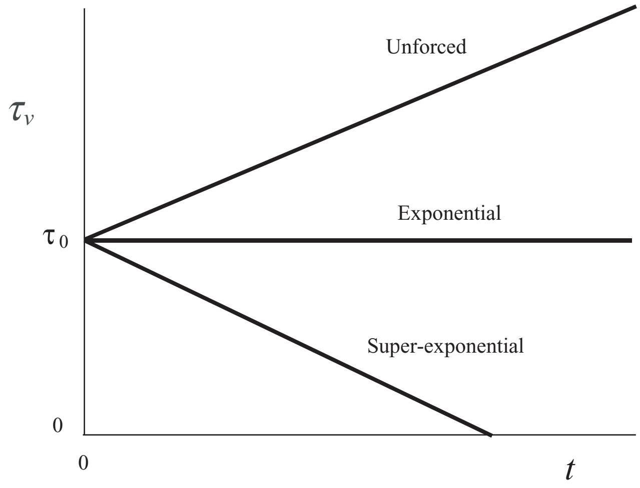

Figure 1. Temporal evolution of the vortex rotation period for self-similar flow.

definition of self-similarity, all self-similar turbulent flows must correspond to one of these three lines, all described by the single parametric equation

$$\tau_v(t) = \tau_0 - \alpha t. \tag{1}$$

Here τ_0 is the rotation period at $t = 0$ and α is an acceleration parameter. We want to find the effect of α on the dimensionless entrainment rate in all generalized laboratory flows.

2.2. Entrainment

The initial guess was that entrainment would decrease with increasing α for all flows. It was qualitatively consistent with the exponential jet as well as a related experiment on the exponential transverse jet (Eroglu and Breidenthal 1998). However, this idea was quickly refuted for the case of Rayleigh–Taylor flow. Unpublished numerical simulations by A Cook and J Greenough demonstrated that Rayleigh–Taylor flow did not fit the mold.

Using dimensional and heuristic arguments, a new theory attempted to account for the peculiar behavior of Rayleigh–Taylor flow (Breidenthal 2003 with different notation). Every canonical flow has some conserved quantity or invariant Q that controls the physics (Cantwell 1981). For example, in the jet, Q is the thrust per unit mass. In the shear layer, Q is the velocity jump ΔU. Since different flows have different dimensions for Q, it may be possible to account for the effect of α in different flows by exploiting the dimensions of Q.

Take the dimensions of Q to be $(\text{length})^m(\text{time})^{-n}$. The dimensions of the dissipation rate per unit mass are $(\text{length})^2(\text{time})^{-3}$. This is proportional to

$$Q^{2/m}\tau_v{}^{-(3-2n/m)}. \tag{2}$$

Since the physically controlling parameter is Q, we expect that every flow may be forced in a self-similar way through

$$Q = Q_0 \exp\{t/(\tau_0 - \alpha t)\}. \tag{3}$$

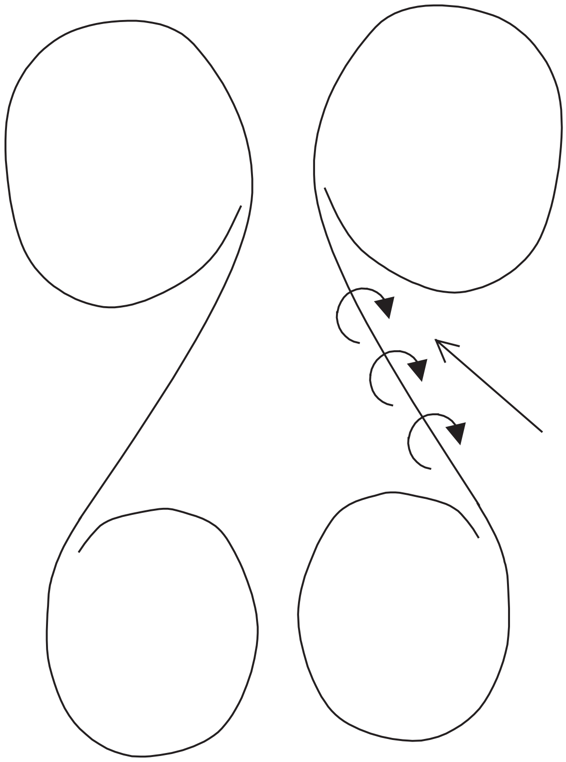

Figure 2. The edge of a plume with increasing buoyancy. Baroclinic torques act to reduce entrainment.

These super-exponential flows are a new class of self-similar turbulence. They are the natural generalization of acceleration on the subset of classical, non-accelerated laboratory flows.

Take D to be the dissipation rate normalized by that of the unforced flow. Based on heuristic grounds, the logarithmic differential of D is assumed to be

$$\mathrm{d}D/D = \mathrm{d}\alpha/\beta, \tag{4}$$

where the natural scaling of acceleration on dissipation is β,

$$\beta = -(3 - 2n/m). \tag{5}$$

It follows that

$$D = \exp\{-(\alpha - \alpha^*)/\beta\}, \tag{6}$$

where $\alpha*$ is the value of α for the unforced flow. Note that if β is negative, dissipation declines with increasing α.

The value of β is negative for every canonical laboratory flow, with two exceptions. For an observer moving down the inertial cascade, β is zero. For Rayleigh-Taylor flow, β is positive (Breidenthal 2003 and 2006). According to the theory, Rayleigh-Taylor flow is unique in that acceleration increases its dissipation rate.

Aside from this dimensional argument, is there another, more physical explanation for the unique behavior of Rayleigh–Taylor? In all other flows, the acceleration vector is essentially parallel to the plane of the vortex sheet. Entrainment tongues are between vortices of the same sign. Figure 2 is a sketch of a two-dimensional plume with increasing buoyancy, related to geophysical flows of latent heat release in clouds (Bhat and Narasimha 1996) and vesiculation in magma flows (Bergantz and Breidenthal 2001). The increasing buoyancy generates

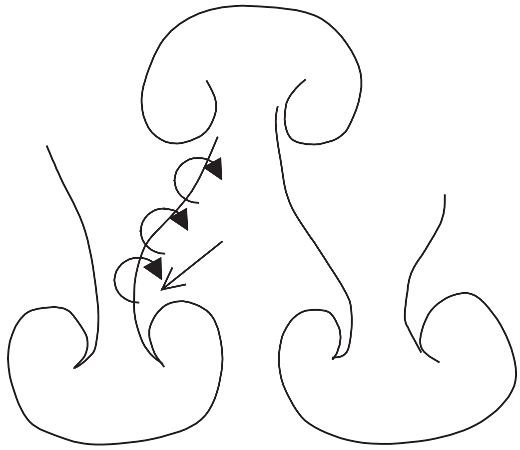

Figure 3. Rayleigh–Taylor entrainment with increasing buoyancy. Baroclinic torques act to increase entrainment.

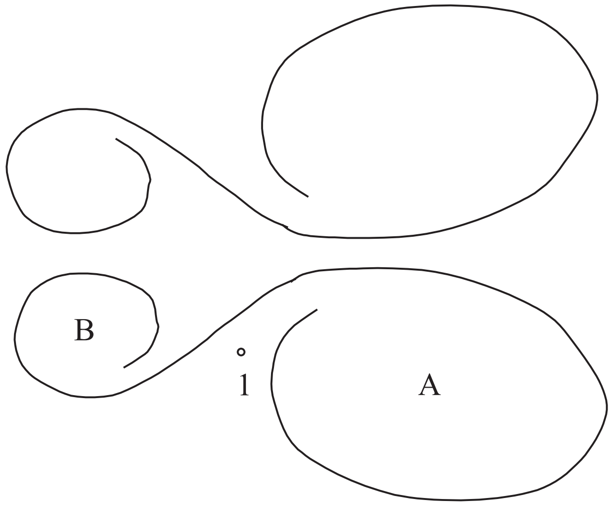

Figure 4. Entrainment in a non-accelerating jet. Point 1 is in the middle of an engulfment tongue.

additional baroclinic torques in the braid region, increasing the vorticity there. The resulting induced velocity from that vorticity acts to reduce the entrainment velocity in Roskho's engulfment tongues (Roshko 1976). Increasing buoyancy reduces entrainment.

On the other hand, the basic geometry for Rayleigh–Taylor is different. There the acceleration vector is essentially orthogonal to the plane of the vortex sheet. Entrainment tongues are between vortices of opposite sign. In figure 3, the baroclinic torques from the acceleration act to increase the entrainment velocity of the engulfment tongues. Acceleration increases entrainment. Because the Rayleigh–Taylor geometry is different, the dependence on acceleration is different.

A final example illustrates the effect of acceleration in the absence of baroclinic vorticity. Consider the uniform density jet in figure 4. Vortex A is older than vortex B and further downstream. If the jet is not accelerating, the circulation of vortex B is on average less than that of A. Suppose Point 1 is closer to the center of Vortex A than to the center of Vortex B. The induced velocity at Point 1 will be dominated by Vortex A, since A is both closer and stronger than B.

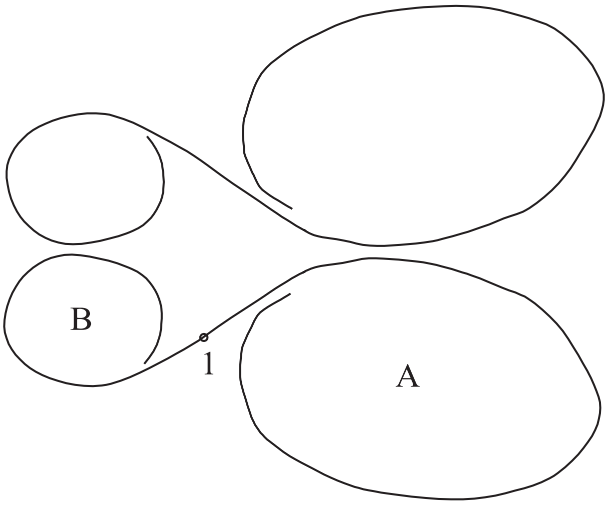

Figure 5. Entrainment in an accelerating jet. Point 1 is now a stagnation point and the engulfment tongue is narrower.

As a consequence, Point 1 resides well within a wide engulfment tongue.

Now imagine that the acceleration parameter α is increased so that the circulation of Vortex B exceeds that of A (figure 5). Under sufficient acceleration, Vortex B will be strengthen with respect to Vortex A such that its induced velocity at Point 1 just nullifies that of Vortex A. In a coordinate frame moving with the vortices, Point 1 then becomes a stagnation point. The surviving engulfment tongue is now much narrower, thereby reducing the average volume flow into the jet. The entrainment velocity has declined due to acceleration. The effect of acceleration on the engulfment tongues can be considered a generalization of the effects of density and velocity ratios on non-accelerating shear flows (Dimotakis 1986).

2.3. Stationary vortices and starting jets

If α is positive, the rotation period vanishes at finite time τ_0/α. As α is increased, this singular time is reduced, so that vortices do not have much time to travel. In the limit of large α, vortices in a jet flow may remain near the nozzle, as seen in analogous exponential transverse jet (Eroglu and Breidenthal 1998). Ironically, such vortices are in a sense stationary, in spite of the violent acceleration. Thus, there is an underlying connection between two seemingly separate phenomena.

It is known that stationary vortices behave differently than nonstationary ones. For example, the entrainment rate across a stratified interface decreases by orders of magnitude when stationary vortices are made nonstationary (Cotel *et al* 1997). The addition of stationary vortices into an initially turbulent boundary layer results in relaminarization over most of the wall (Balle and Breidenthal 2002, Dawson 2005, Bauer 2006).

When a vortex is near some surface, one can define an intrinsic velocity ratio of the vortex, the ratio of the rotational to the translational speed with respect to the surface (figure 6). Cotel and Breidenthal (1994, 1997) identified this initially in stratified flow, calling it a vortex 'persistence' parameter. According to their argument, this parameter should be important anytime a vortex is near any definable surface,

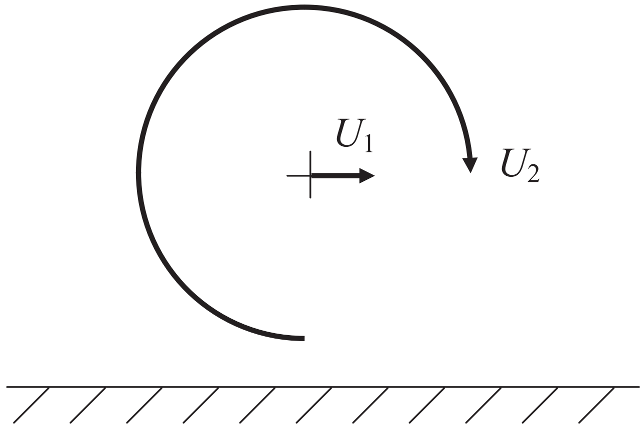

Figure 6. The intrinsic velocity ratio U_2/U_1 of a vortex near any surface (Cotel).

such as a stratified interface, a solid wall, or an iso-vorticity surface of another vortex. As noted by Gharib (1995, private communication), the persistence parameter is also equivalent to the formation number in starting jets (Gharib *et al* 1998). They found that there was a critical value of formation number corresponding to the point at which no more vorticity could be accommodated into the starting vortex.

Since acceleration reduces the dissipation and entrainment rates in the jet, the critical formation number should increase in accelerating jets. This qualitative trend has already been observed in non-self-similar jets (Shusser *et al* 2006, Yu *et al* 2007). According to the theory, the critical formation number should increase exponentially with α for the self-similar jet.

3. Conclusions

A new class of self-similar, accelerating flows is proposed as a generalization of classical, non-accelerating turbulence. A new theory analyzes the general behavior of acceleration on entrainment. According to the theory, the normalized entrainment velocity is an exponential function of an acceleration parameter. Acceleration reduces the entrainment rate in all flows except for Rayleigh–Taylor, whose geometry is unique.

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