Super-exponential magnetic confinement

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It is proposed that a magnetic field that increases rapidly in time may increase the stability of magnetic confinement and reduce undesirable wall fluxes in fusion applications. The rate of increase in the strength of the magnetic field is parameterized by an acceleration parameter α for the self-similar case of super-exponential time dependence. The fluxes are predicted to decline exponentially with increasing α.

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Introduction.----In a classical fluid at large Reynolds number, the flow is typically turbulent. Counter-intuitively, the fluxes of mass, momentum, and energy due to turbulent entrainment appear to decline during strong acceleration. By acceleration, we mean rapid increase in the rotation rate of the largest eddies. For example, the flame length in a turbulent jet is increased when the nozzle speed increases exponentially [1,2]. The normalized entrainment velocity determines the flame length. An increase in flame length implies a reduction in normalized entrainment rate. Another flow also seems to behave in this manner, the exponential transverse jet [3].

A model has been developed to describe the effects of acceleration [4]. In the absence of acceleration, the rotation period of a vortex $\tau_v(t)$ is proportional to its chronological age t. Under forcing, however, τ_v becomes the e-folding time of the forcing function τ_e

$$\tau_v = \tau_e. \tag{1}$$

For the flow to remain self-similar, τ_v must be a linear function of t

$$\tau_v(t) = \tau_0 - \alpha t, \tag{2}$$

where τ_0 is the period at the arbitrary time t = 0. The coefficient α is the acceleration parameter.

For ordinary, unforced turbulence, $\alpha < 0$. If τ_v is a constant in time, as in the case of the exponential jet, then $\alpha = 0$ and all vortices will rotate with the same period [2]. If $\alpha > 0$, the vortex period will decrease in time, vanishing at time $t = \tau_0 / \alpha$. The flow becomes singular then. We will limit our discussion to times less than this.

In the model, a heuristic argument uses dimensional analysis to conclude that the effect of acceleration on dissipation and hence entrainment depends not only on the acceleration parameter α, but also on the dimensions of the conserved quantity Q of the flow. For all canonical laboratory flows, the dissipation rate declines exponentially with α, except for Rayleigh-Taylor. This implies that acceleration in inertial-confinement fusion will increase the entrainment rate, unfortunately. However, it may be possible to reduce the fluxes in magnetic-confinement fusion by rapidly increasing the magnetic field.

Magnetic-confinement fusion.---In the absence of net charge, the magnetic field $\vec{B}(\vec{x},t)$ does not exert a Lorentz force on a fluid element if

$$\nabla \times \vec{B} = \lambda \vec{B}, \tag{3}$$

where λ is an eigenvalue corresponding to the inverse length scale of the field [5].

Since this equation contains no time derivatives, it can be satisfied by a magnetic field $\vec{B}(\vec{x},t)$ of the appropriate spatial distribution $\vec{B}_x(\vec{x})$ multiplied by a super-exponential function of time

$$\vec{B}(\vec{x},t) = \vec{B}_x(\vec{x}) \exp\left(\frac{t}{\tau_0 - \alpha t} \right). \tag{4}$$

Conserved quantity.---It is not clear what the conserved quantity Q should be for this problem. One guess is to assume that Q is simply $1/\lambda$, the relevant length scale. In that case, the model predicts that the dissipation rate per unit mass D is

$$D = D^* \exp\left[\left(\alpha^* - \alpha \right) / \beta \right], \tag{5}$$

where the starred symbols correspond to the unforced case and $\beta = 3$. If correct, operating such a magnetic-confinement machine at large values of α would increase the stability of the magnetic field and reduce the wall fluxes.

Of course, the field would not be steady state, and eventually the acceleration must end. One could imagine a sawtooth function in time for the magnetic field strength at every point in space, in which the strength increased rapidly in time and then returned to some initial low value, and then the cycle repeated.

Conclusions.---It is proposed that a rapid increase in the strength of the magnetic field may reduce the wall fluxes and increase the performance of magnetically-confined fusion devices. A device pursuing this strategy might generate a magnetic field strength that everywhere increases super-exponentially for an interval, drops back to a low value, and then begins another cycle.

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