Comparisons of Two-Fluid Plasma Models

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1 Introduction

Two-fluid models of plasmas are explored. Single fluid models of plasmas which include Ideal- and Resistive-Magnetohydrodynamics (MHD) are frequently used to simulate plasma behavior but they often fail to capture two-fluid effects. Two-fluid effects become significant when characteristic spatial scales are small compared to the ion skin depth and the characteristic time scales are short compared to the inverse ion cyclotron frequency. The motivation here is to explore higher fluid models of plasmas namely the two-fluid plasma model and the asymptotically approximated two-fluid model, Hall-MHD.

2 Plasma fluid models

2.1 Two-fluid plasma model

The full two-fluid plasma model is described using the Euler equations for the electron and ion fluids. This includes the continuity, momentum and energy equations for both fluids. The electric and magnetic fields are described using a full set of Maxwell's equations[1]. In this five moment, two-fluid plasma model, the characteristic speeds include the fluid speeds of sound and the speed of light. Resolving the speed of light, which is the fastest characteristic speed in the system, allows small spatial- and temporal-scale physics to be captured. The speed of light restricts the time step.

2.2 Hall-MHD

Three asymptotic approximations are applied to the full two-fluid model described previously. These include setting the speed of light to infinity, neglecting electron inertia and assuming quasi-neutrality. The ion fluid is still described by the full set of Euler equations. The continuity equation of the electron fluid is eliminated due to the quasi-neutrality assumption. The momentum equation of the electron fluid breaks down due to the massless electron assumption and results in the generalized Ohm's law. The electron pressure gradient is assumed to equal the ion pressure gradient.

Since infinite speed of light is assumed, the time step for Hall-MHD is restricted by the whistler wave which grows quadratically without bound. This unbounded whistler wave results from neglecting electron inertia. Resolving this whistler wave makes Hall-MHD more computationally challenging to solve compared to the two-fluid model as it requires a very restrictive time step.

3 Numerical method and Computing Resources

3.1 Numerical method used

The discontinuous Galerkin method[2], a finite element method, is used to evolve the solution in space. For the problems in this paper, a 2^{nd} order spatial scheme is used. The solution is evolved in time using a 3^{rd} order Runge-Kutta time integration scheme. The Runge-Kutta discontinuous Galerkin (RKDG) method is used to solve the full set of hyperbolic equations with source terms for the two-fluid model. For Hall-MHD, the hyperbolic fluid equations and Faraday's laws are solved using the RKDG method. Central differencing is used to solve the generalized Ohm's law to get the electric fields.

The time step of the RKDG method depends on the spatial order used which makes it slower and more restrictive than other methods like the high resolution wave propagation method[3], a finite volume method. The two-fluid plasma model has been implemented with the high resolution wave propagation method in Ref.[4]. However, the RKDG method can run at higher orders of accuracy while the wave propagation method is 2^{nd} order accurate.

3.2 Code and computing resources used

WARPX (Washington Approximate Riemann Plasma code), a C++ code developed at the University of Washington, is used for all the results presented in this paper. The code is explicit and employs finite volume and finite element algorithms. Being an explicit code, it scales well with parallel processing. A parallel array object is used to allow for parallel N-dimensional arrays in the code. A parallelization algorithm is used to internally breakup the domain into the desired number of processors while attempting to keep the areas within each processor as close as possible. The results in this paper were run using upto 200 processors on the Maui High Performance Computing Center's (MHPCC) Jaws system.

The data output is in hdf5 format so parallel versions of hdf5 are required to be compiled with the version of mvapich that is used. Hdf5, mvapich and a software construction tool, scons, were made available on the Jaws system which were the only other dependencies for WARPX besides the standard C++ compilers.

Using the MHPCC resources enabled a number of memory leaks in WARPX to be eliminated that were on the order of only a few bytes per time step so it was necessary to run large simulations in order to detect them. Tools such as valgrind were used to profile the code and determine the locations of the leaks.

Matplotlib is used for visualization of all the 1-dimensional and 2-dimensional results. VisIt is used for 3-dimensional visualization. All the plots shown here include data from the quadrature points for the RKDG method, i.e. for a 2^{nd} order 128×128 cell RKDG solution, the data is plotted at the effective resolution of 256×256.

4 Results

The two-fluid plasma model is compared to Hall-MHD for applications of magnetic reconnection, the 2-dimensional axisymmetric Z-pinch and the 3-dimensional Z-pinch.

4.1 Magnetic Reconnection

A current sheet is initiated with an in-plane current in the x-direction and an in-plane magnetic field in the x-direction. A small initial perturbation is applied to the in-plane magnetic fields in the x- and y-directions to initiate magnetic reconnection. The results obtained are comparable to previously published results[5]. Figure 1 shows magnetic reconnection when using the two-fluid plasma model and Fig. 2 shows the result obtained using Hall-MHD. The differences lie in the formation of an island in the center of the domain for the two-fluid case that does not form in the Hall-MHD case.

The two-fluid and Hall-MHD models are both able to simulate magnetic reconnection with slight differences. The two-fluid and Hall-MHD simulations are both run out to an ion cyclotron time of 60. The two-fluid model takes about 30 seconds of CPU time per advance whereas Hall-MHD takes 450 seconds of CPU time per advance where one advance is the computation time between writing output files. The two-fluid model was run using 8 processors whereas Hall-MHD was run using 32 processors on Jaws for this problem. Both models use 128×64 cells with 2^{nd} order RKDG.

4.2 Axisymmetric Z-pinch in 2-dimensions

A 2-dimensional axisymmetric Z-pinch problem is also compared between the models. The Z-pinch is initialized as described in Refs.[6] and [7].

Figures 3 and 4 both show the growth of the short wavelength lower hybrid drift instability. The ion Larmor radius, $r_{Li} = v_{thi}/\omega_{ci}$, needs to be resolved spatially in the domain to be able to see the lower hybrid drift instability. The parameters and grid resolution are chosen accordingly to capture the two-fluid effects. As a result of this, it takes more computational effort to complete the simulation than if it were in the single fluid regime. Here v_{thi} is the ion thermal velocity and ω_{ci} is the ion cyclotron frequency. This problem was run using 8 processors for the two-fluid model and took 160 seconds of CPU time for one advance on a 128×128 cell grid with 2^{nd} order

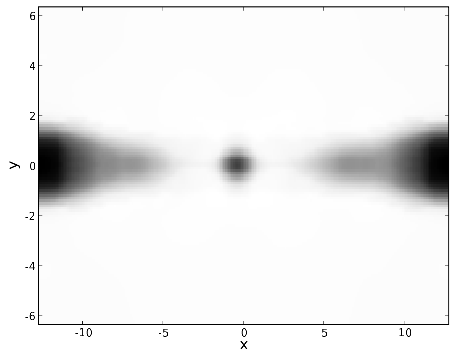

Figure 1: The evolution of ion density at a time of 20 ion cyclotron times is shown for the two-fluid model. This solution is obtained using time steps of about 3×10^{-2}.

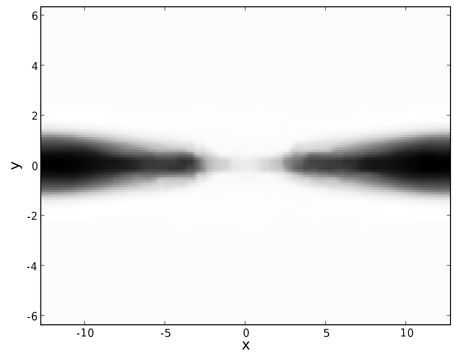

Figure 2: The evolution of ion density at a time of 20 ion cyclotron times is shown for the Hall-MHD model. This solution is obtained using time steps of about 1.2×10^{-3}.

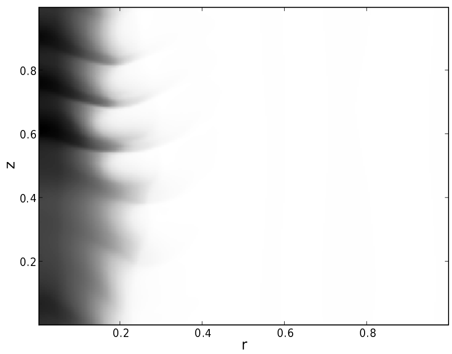

Figure 3: Ion density for the two-fluid model for the axisymmetric Z-pinch after 1.75 Alfven transit times. The lower hybrid drift instability is observed. The two-fluid time steps are about 1×10^{-3}.

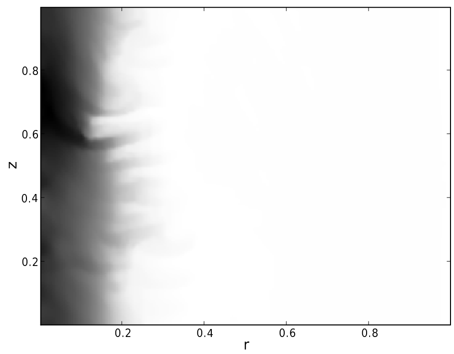

Figure 4: Ion density for Hall-MHD for the axisymmetric Z-pinch after 1.75 Alfven transit times. The lower hybrid drift instability is observed. The Hall-MHD time steps are about 2×10^{-5}.

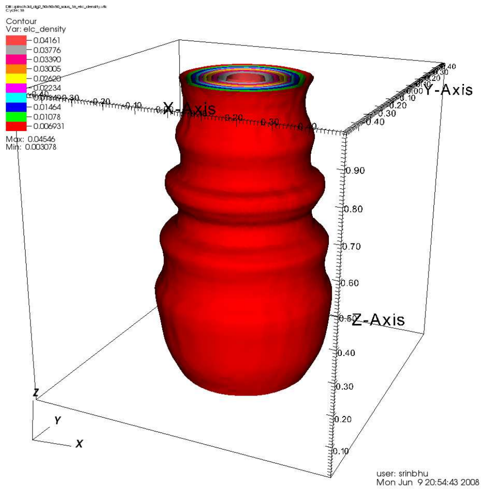

Figure 5: Ion density for the two-fluid model for the 3-dimensional Z-pinch with a sausage instability perturbation after 2 Alfven transit times. The lower hybrid drift instability is observed.

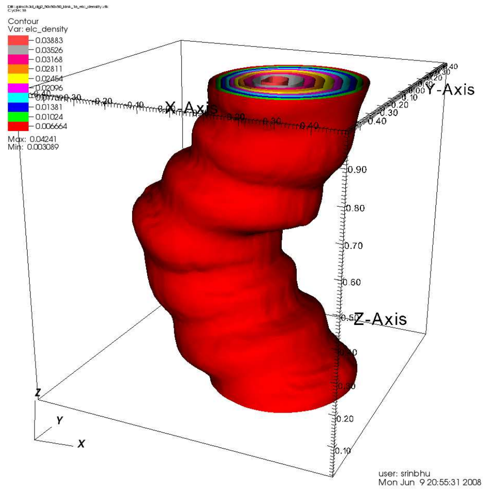

Figure 6: Ion density for Hall-MHD for the 3-dimensional Z-pinch with a kink instability perturbation after 2 Alfven transit times. The lower hybrid drift instability is observed.

RKDG. Using the same resolution for Hall-MHD with 64 processors on Jaws, it took $5,900$ seconds of CPU time for one advance. Hall-MHD is extremely computationally intensive and high performance computing is necessary for small spatial scale and short temporal scale two-fluid physics to be captured using Hall-MHD.

4.3 Z-pinch in 3-dimensions

The Z-pinch has been simulated in 3-dimensions using the two-fluid plasma model. Due to the extremely large computational effort, this problem has not yet been simulated with Hall-MHD. A grid with a resolution of $50\times50\times50$ cells is used. The initializations of the fluid and electromagnetic variables are the same as the 2-dimensional case but extended out to 3-dimensions.

Figures 5 and 6 show the short wavelength modes form on top of the single wavelength sausage and the kink instability perturbations that were applied to the initializations. These simulations are run out to an Alfven transit time of 5 and the plots shown are at an Alfven transit time of 2. Just as with the 2-dimensional case, the ion Larmor radius needs to be resolved spatially in the 3-dimensional case as well in order to see the lower hybrid drift instability. Each of the simulations takes about 300 hours of CPU time to run out to 5 Alfven transit times. This is a low resolution simulation and higher resolutions might be necessary to see more details in the solution. Higher orders of the RKDG method run with higher grid resolutions will most certainly require high performance computing resources such as Jaws for 3-dimensional problems using both the two-fluid plasma model and Hall-MHD.

5 Conclusions

The two-fluid and Hall-MHD models have been compared for problems involving two-fluid physics such as magnetic reconnection, lower hybrid drift instability evolution in a 2-dimensional axisymmetric Z-pinch and a 3-dimensional Z-pinch. Both models are able to capture two-fluid effects that are often missed by simpler single fluid models. However, Hall-MHD takes between 15 to 50 times more computational effort than the two-fluid model for the same grid resolution for the 2-dimensional problems explored here. For 3-dimensional problems, Hall-MHD would require enormous computational effort to capture two-fluid effects. The 3-dimensional Z-pinch studied here can be explored using higher grid resolutions and higher spatial orders for the RKDG method to fully and accurately simulate the growth of the lower hybrid drift instability. This would require extensive use of supercomputing resources such as the MHPCC Jaws system. Higher fluid models such as Hall-MHD and the two-fluid plasma model capture small-scale physics that are often missed when using simpler single fluid models. Fluid models simulate plasma behavior with less noise and less computational effort as compared to particle codes. The ability to capture two-fluid effects and simulate small-scale physics can be applicable to real experiments such as Hall thrusters, Helicon thrusters, field reversed configurations and more alternative fusion concepts that are of interest to the DoD.

Acknowledgments

The work presented in this paper was supported by AFOSR Grant No. FA9550-05-1-0159. The MHPCC Jaws system was used to generate the results presented here. The authors wish to thank Ammar Hakim for developing the parallel architecture in WARPX that has been extensively used here. The authors also wish to thank John Loverich for helpful discussions that enabled Hall-MHD to be successfully implemented with the discontinuous Galerkin method.

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Summary

Introduction

Advanced physics fluid models of plasmas are explored. Euler equations are used to model ion and electron fluids, while Maxwell's equations are used to model the electric and magnetic fields. This constitutes the ideal full two-fluid plasma model. Two-fluid effects become significant when the characteristic spatial scales are small compared to the ion skin depth and the characteristic time scales are short compared to the inverse ion cyclotron frequency which could justify the use of two-fluid plasma models over single fluid models.

Objective

Asymptotic approximations are applied to the full two-fluid plasma system. The approximations include ignoring electron inertia, setting the speed of light to infinity, and ensuring charge neutrality. Applying all three approximations together leads to the Hall-MHD model. The motivation here is to compare the asymptotic two-fluid models to the full two-fluid model to study the physics that is lost by applying the approximations in addition to comparing the simplicity of implementation of the models.

Methodology

Using the Hall-MHD model causes the Whistler wave in the system to grow without bound which largely affects the time step required to resolve all the physics in this system. The asymptotically approximated model, Hall-MHD, is compared to the two-fluid plasma model. The physical effects that are gained or lost from applying these approximations are studied to determine the realm of applicability of each of these fluid models.

The WARPX (Washington Approximate Riemann Plasma) code developed at the University of Washington is used to perform the simulations. The numerical algorithm used for these simulations is the Runge-Kutta discontinuous Galerkin method, a finite element method. Hall-MHD includes a central differencing algorithm used to compute the electric field.

Results

The two-fluid effects of Hall-MHD are compared to the full two-fluid plasma model through applications to problems such as magnetic reconnection and the axisymmetric Z-pinch. Hall-MHD uses greater computational effort compared to the two-fluid plasma model for the same grid resolution due to the unbounded Whistler wave.

Significance to DoD

The High Performance Computing machines are used for multi-dimensional modeling of the two-fluid plasma systems using high spatial orders of accuracy and high resolution in order to capture the small-scale physics effectively. The ability to capture two-fluid effects and small-scale physics can be applicable to DoD projects such as Hall thrusters, Helicon thrusters, Field Reverse Configurations and more.