A Comparison between the Two-fluid Plasma Model and Hall-MHD for Captured Physics and Computational Effort[1]

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Abstract & Motivation

- The two-fluid plasma model is studied and compared to the asymptotic model, Hall-MHD.
- Three asymptotic approximations are applied to the full two-fluid plasma model to obtain Hall-MHD namely, charge neutrality, infinite speed of light and negligible electron inertia.
- Two-fluid effects become significant when the characteristic spatial scales are on the order of the ion skin depth and the characteristic time scales are on the order of the inverse ion cyclotron frequency. The Hall and diamagnetic drift terms capture the two-fluid physics.
- Hall-MHD is compared to the full two-fluid plasma model for the physics that is captured as well as the computational effort.

Abstract & Motivation

- Artificially decreasing the ion-to-electron mass ratio in the two-fluid plasma model captures all the Hall-MHD physics while using less computational effort.
- Likewise, artificially decreasing the ratio of the speed of light to the Alfven speed in the two-fluid plasma model also captures Hall-MHD with less computational effort.
- The two-fluid model provides the solution obtained by Hall-MHD using less computational effort and without the need for artificial dissipation.
- Simulations of the electro-magnetic plasma shock, collisionless magnetic reconnection, axisymmetric Z-pinch and field reversed configuration are explored and the results are compared between the models.

Two-Fluid Plasma Model

Euler equations are used for ion and electron fluids denoted by subscript s.

$$
\begin{aligned}
\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) &= 0 \\
\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + \nabla p_s \mathbf{I}) &= \frac{\rho_s q_s}{m_s}(\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) \\
\frac{\partial \epsilon_s}{\partial t} + \nabla \cdot ((\epsilon_s + p_s)\mathbf{u}_s) &= \frac{\rho_s q_s}{m_s}\mathbf{u}_s \cdot \mathbf{E} \\
\epsilon_s &\equiv \frac{p_s}{\gamma - 1} + \frac{1}{2}\rho_s u_s^2.
\end{aligned}
$$

Maxwell equations are used to evolve the electric and magnetic fields.

$$
\begin{aligned}
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0 \\
\frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = \mu_0 \sum_{\mathbf{s}} \frac{\mathbf{q_s}}{\mathbf{m_s}} \rho_{\mathbf{s}} \mathbf{u_s}
\end{aligned}
$$

Hall-MHD Equation System by Applying Asymptotic Approximations

- Euler equations for ions and Faraday's law same as two-fluid model.
- Quasi-neutrality assumption eliminates electron continuity.
- Neglecting electron inertia reduces electron momentum to Generalized Ohm's law

$$nq_e\mathbf{E} = \nabla p_e - \mathbf{J}_e \times \mathbf{B}$$

- Infinite speed of light reduces Ampere's law to

$$\mathbf{J} = \frac{1}{\mu_0}\nabla \times \mathbf{B} \text{ where } \mathbf{J} = \mathbf{J}_i + \mathbf{J}_e$$

- Hall-MHD model using electron energy equation and using $\nabla p_e = \nabla p_i$ is studied

Ideal-MHD Equation System

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u} + P\bar{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} + \frac{B^2}{2\mu_0}\bar{\mathbf{I}}) = 0$$

$$\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left[\left(\varepsilon + P + \frac{B^2}{2\mu_0} \right) \mathbf{u} - \frac{(\mathbf{B} \cdot \mathbf{u})}{\mu_0} \mathbf{B} \right] = 0$$

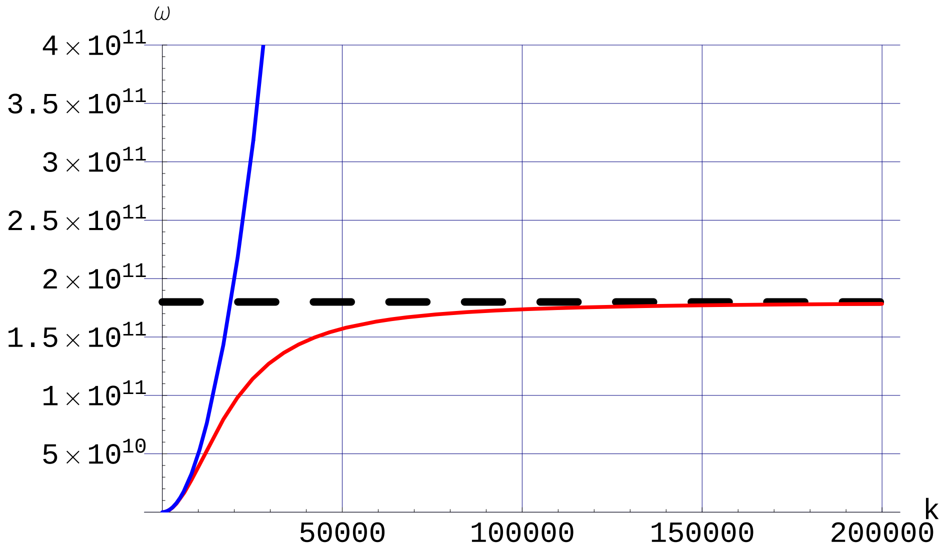
$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{B} - \mathbf{B}\mathbf{u}) = 0$$

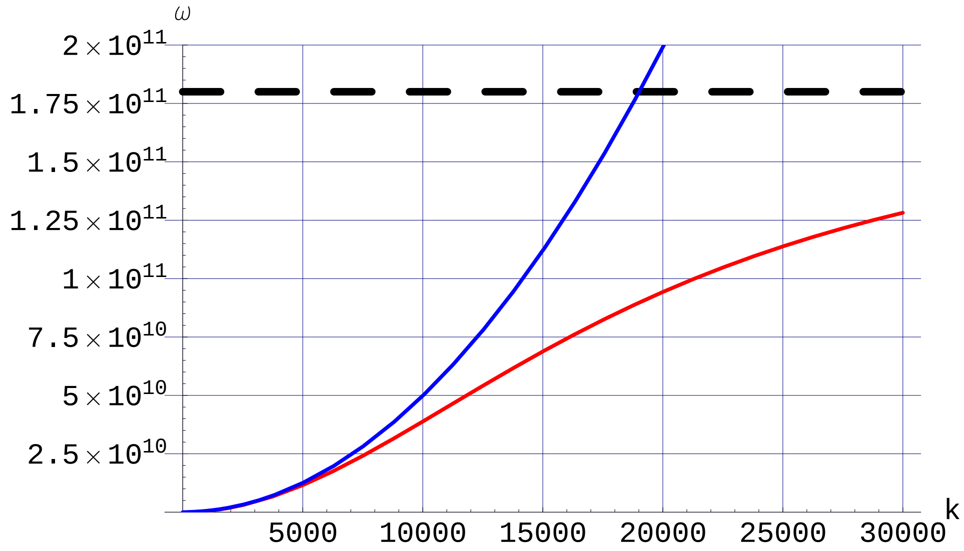
The Ideal-MHD model does not contain the Hall and the diamagnetic drift terms that are present in Hall-MHD.

Computing Resources & Numerical Method

- WARPX (Washington Approximate Riemann Plasma) code
- Runge-Kutta, discontinuous-Galerkin (RKDG) method used here
- For Hall-MHD, the current (from reduced Ampere's law) and the electric field (from Ohm's law) are introduced as auxiliary variables using a basis function expansion with the RKDG method.
- Floor values are used for density and pressure especially for Hall-MHD to prevent negative pressure errors which occur more often when using the Hall-MHD model with the electron energy equation.
- Maui High Performance Computing Center's Jaws system is used with 1000 CPU hours per simulation.

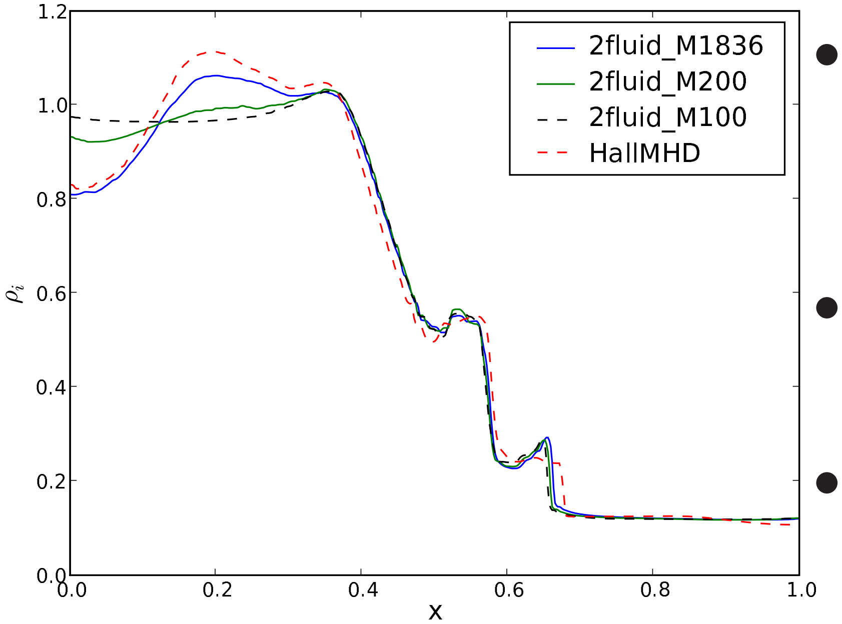
Whistler Wave Dispersion Relation





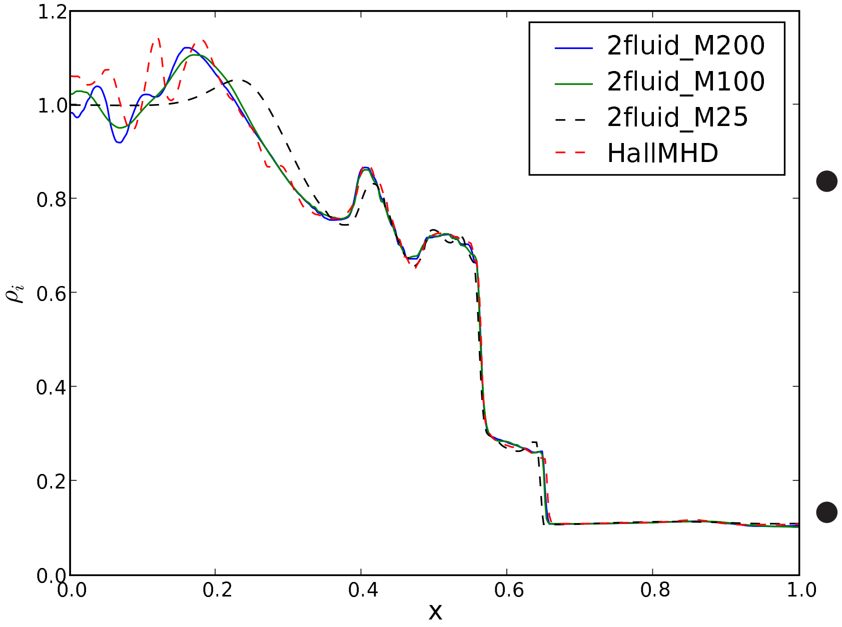
- Right plot is an expanded version of the left plot
- Black-dashed: electron cyclotron frequency
  Blue: Hall-MHD whistler wave
  Red: Two-fluid plasma model whistler wave
- Hall-MHD whistler wave grows without bound while two-fluid whistler wave asymptotes at electron cyclotron frequency.

Electro-Magnetic Shock Comparisons for $r_{Li} = 7 \times 10^{-1}$



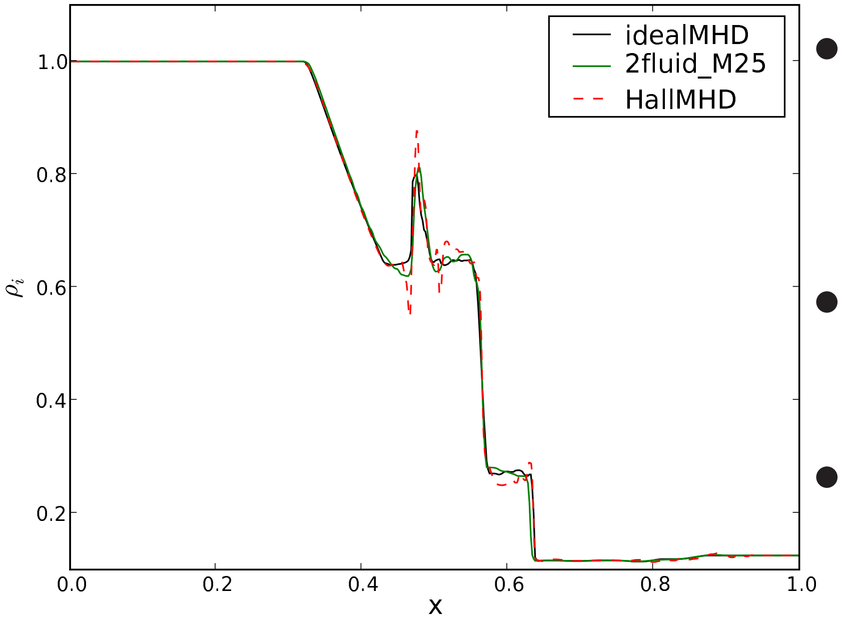
- MHD limit has $r_{Li} = 0$
- Two-fluid and Hall-MHD ion density after 10 characteristic transit times
- Two-fluid model with several mass ratios
- Hall-MHD takes 135 times more computational effort than the two-fluid model.

Electro-Magnetic Shock Comparisons for $r_{Li} = 7 \times 10^{-2}$



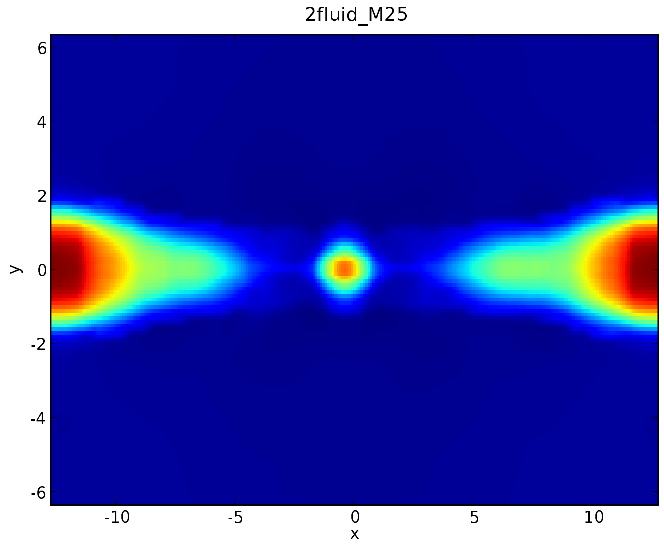
- Two-fluid and Hall-MHD ion density after 10 characteristic transit times
- Two-fluid model with several mass ratios, using higher mass ratios becomes too stiff in this regime
- Hall-MHD takes 14 times more computational effort than the two-fluid model.

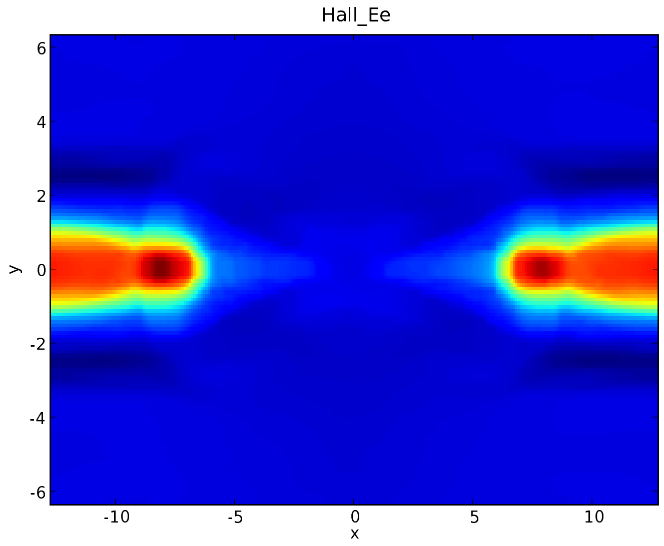
Electro-Magnetic Shock Comparisons in Ideal MHD regime



- For two-fluid and Hall-MHD models, $r_{Li} = 7 \times 10^{-4}$
- Two-fluid, Hall-MHD and ideal-MHD ion density after 10 characteristic transit times
- Two-fluid model becomes stiff in this ideal-MHD regime
- Hall-MHD takes less computational effort than the two-fluid model in this regime and ideal-MHD takes the least.

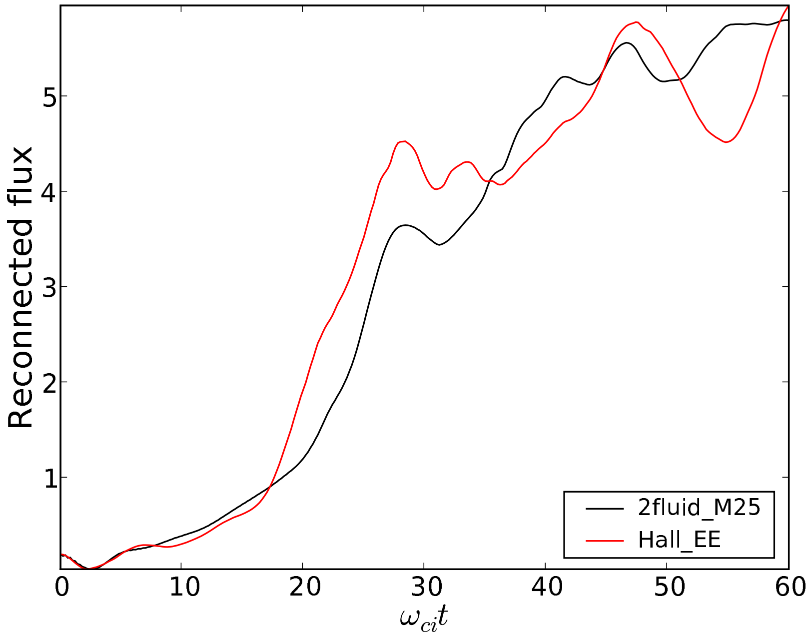
Magnetic Reconnection GEM Challenge Problem





- Left: Two-fluid solution of ion density at $\omega_{ci}t = 20$
  Right: Hall-MHD solution of ion density at $\omega_{ci}t = 20$
- Note island formation in two-fluid model that moves and merges to the left of the domain.
- Two-fluid model uses $\frac{m_i}{m_e} = 25$ and light speed, $c \approx 10V_{Alfven}$

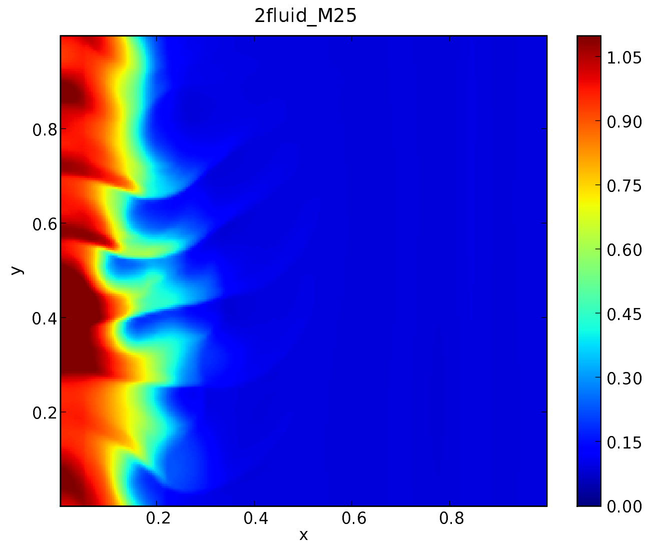
Magnetic Reconnection - Reconnected Flux

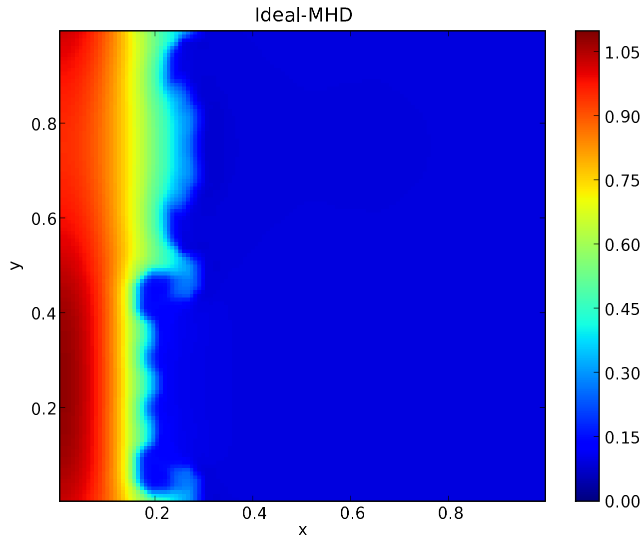


- Two-fluid and Hall-MHD with electron energy equation reconnection rates consistent with previous literature[a].
- Hall-MHD takes 15 times more computational effort than the two-fluid model for a comparable solution.

[a] Shay et al., Journal of Geophysical Research, 2001

Axisymmetric Z-pinch Instabilities

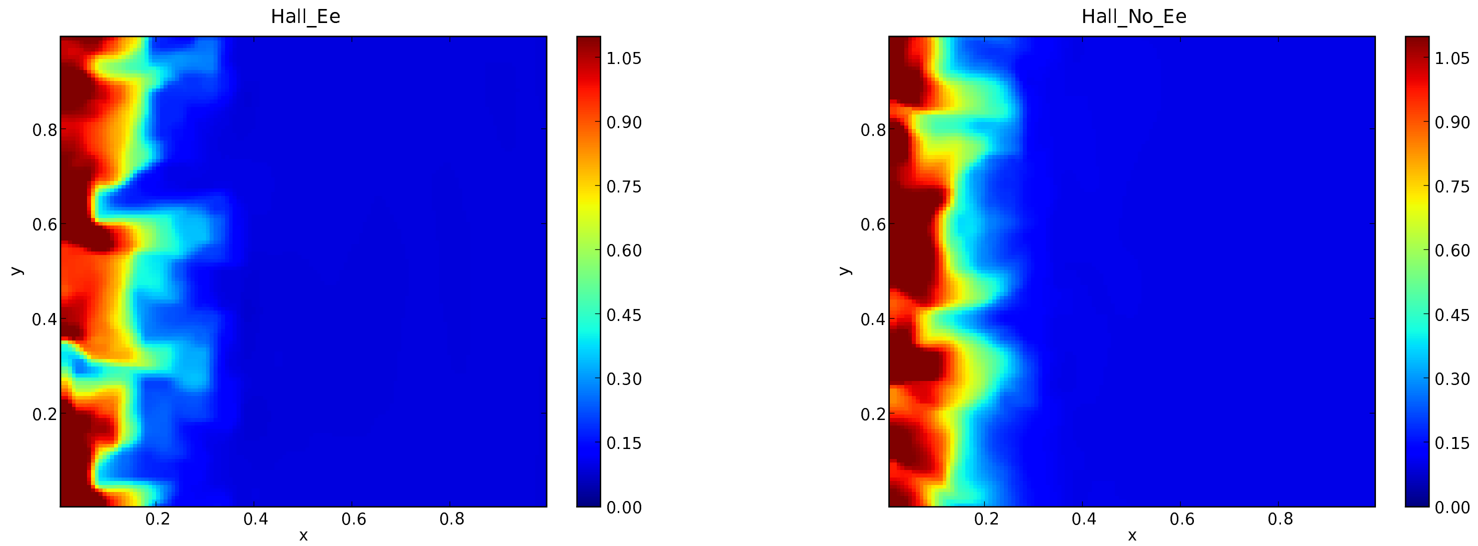




- Left: Two-fluid ion density after 2.5 Alfven transit times
  Right: Ideal-MHD ion density after 2.5 Alfven transit times
- Initializations same as Loverich et al.[b] with $\frac{R_{Li}}{R_p} \approx 3$, $\frac{m_i}{m_e} = 25$ and $c \approx 16V_{Alfven}$, with single wavelength perturbation

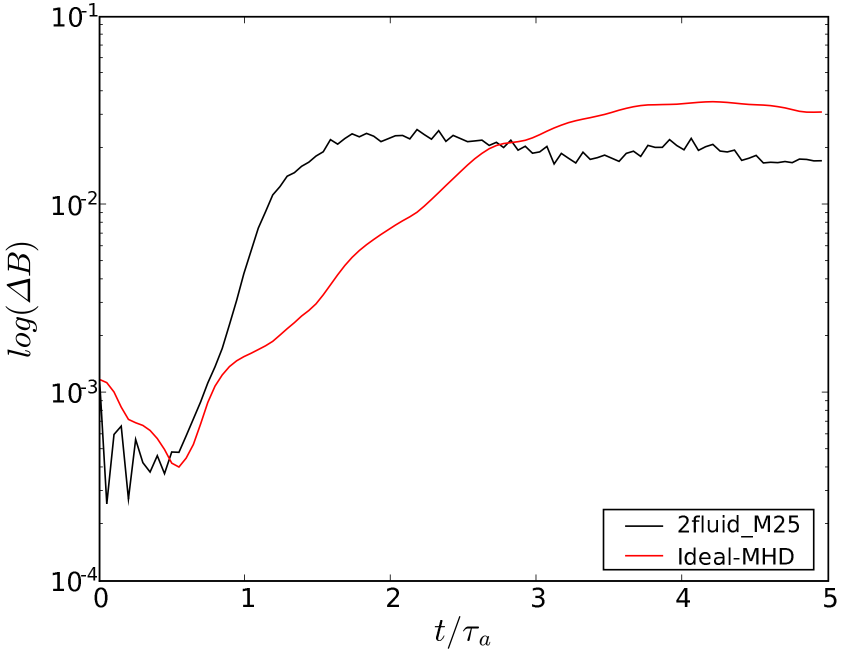
[b] Loverich & Shumlak, Physics of Plasmas, 2006

Axisymmetric Z-pinch Instabilities



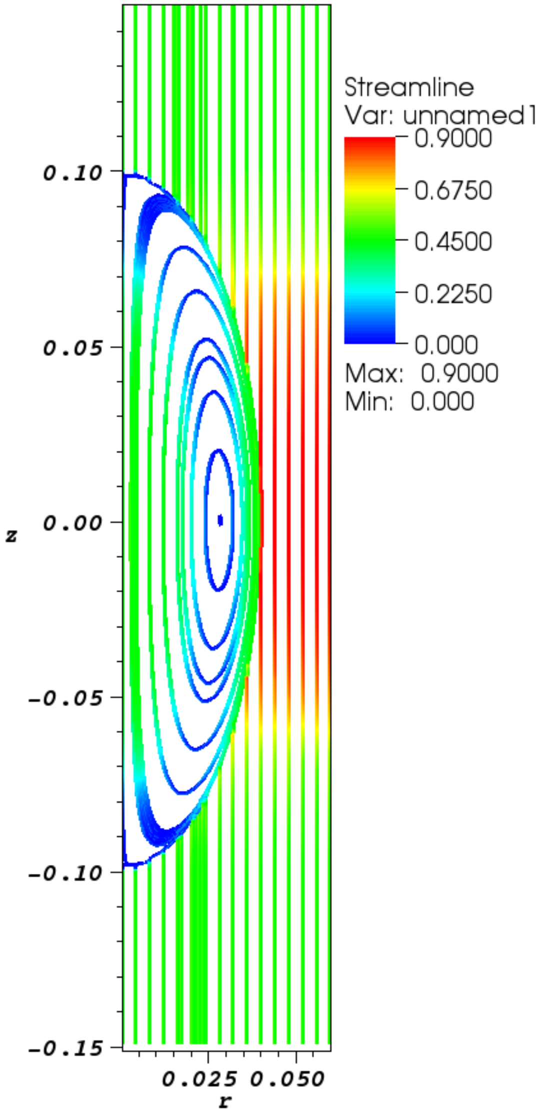
- Left: Hall-MHD with electron energy, ion density after 2.5 Alfven transit times
  Right: Hall-MHD with $\nabla p_e = \nabla p_i$, ion density after 2.5 Alfven transit times
- The two-fluid and Hall-MHD models capture the lower hybrid drift instability whereas ideal-MHD does not.

Axisymmetric Z-pinch Instability Growth Rates



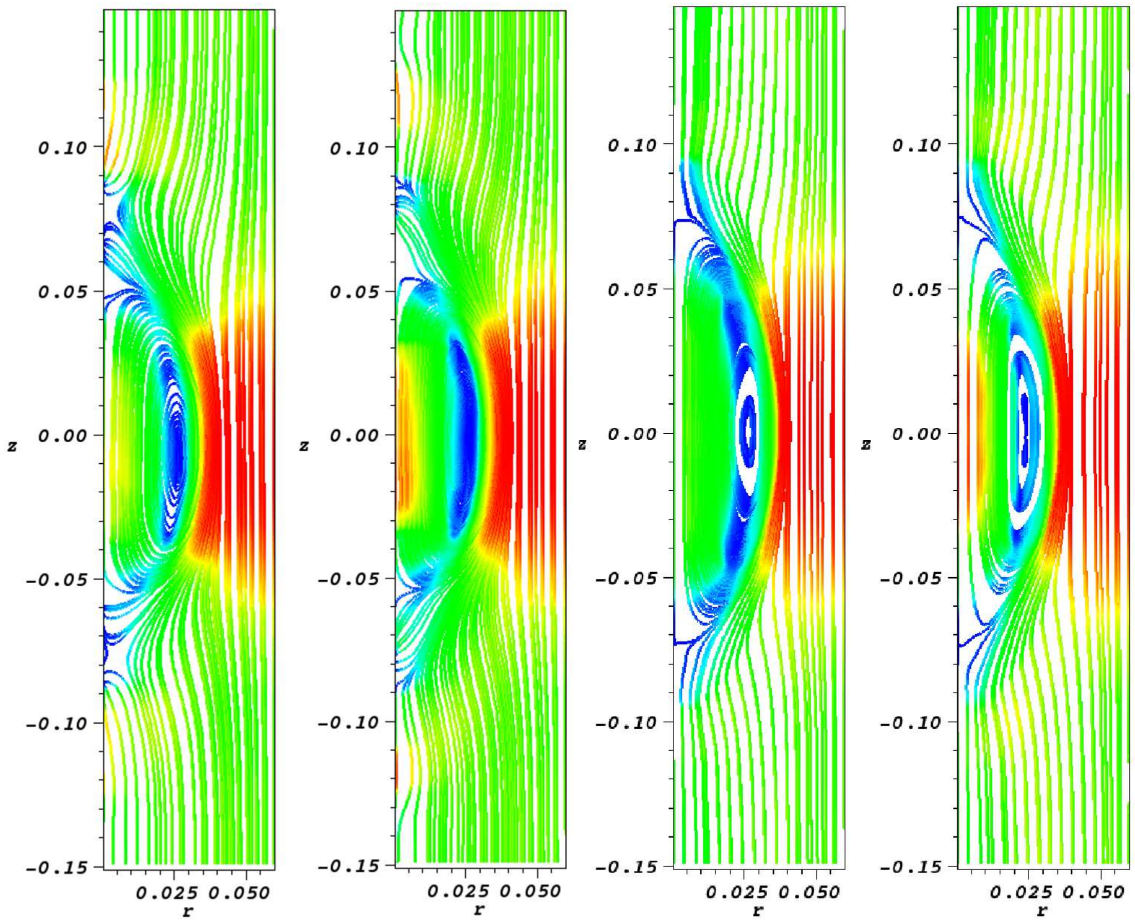
- Two-fluid and ideal-MHD growth rates for the instability
- The instability grows faster for the two-fluid model than for ideal-MHD. This is the lower-hybrid drift instablity that is not captured by ideal-MHD.
- Hall-MHD requires 35 times the computational effort of the two-fluid model for this problem.

Hill's Vortex FRC Initial Condition



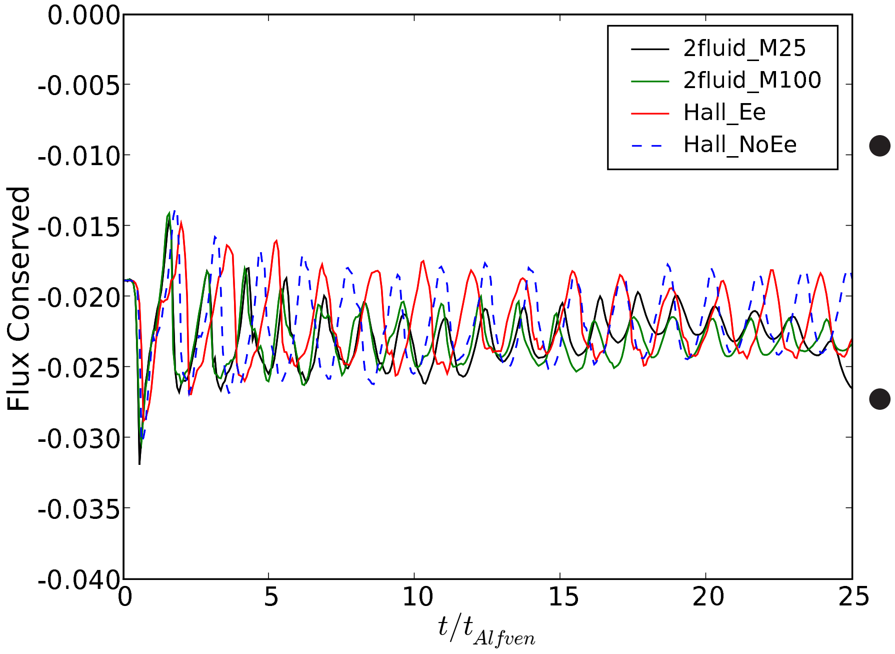
- Initial condition of magnetic flux contours
- Initial peak density $\approx 3.3 \times 10^{21}/m^3$
- Initial $T_i = T_e = 100eV$
- Kinetic parameter, $s \approx 4$
- For two-fluid, $c \approx 22 V_{Alfven}$
- For large $\frac{m_i}{m_e}$ for two-fluid model, electron plasma frequency can be more restrictive than the light speed for setting a time step.

Hill's Vortex Flux Contours at $t = 13t_{Alfven}$



Left to Right: Two-Fluid $\frac{m_i}{m_e} = 25$, Two-Fluid $\frac{m_i}{m_e} = 100$, Hall-MHD with electron energy, Hall-MHD with $\nabla p_e = \nabla p_i$

Hill's Vortex Flux Conservation



- Conserved flux between null-point and axis for the two-fluid and Hall-MHD models agree
- Oscillations occur as plasma bounces back and forth, no explicit dissipation is present.
- Hall-MHD requires 8-to-30 times the computational effort of the two-fluid model for this problem depending on the density/pressure floor that is chosen.

Conclusions

- Both the two-fluid plasma model and Hall-MHD are able to capture two-fluid physics such as magnetic reconnection and the short-wavelength lower hybrid drift instability.
- Hall-MHD requires between 8 and 50 times the computational effort of the two-fluid model for 1- and 2-dimensional problems.
- No explicit dissipation is added for any of the models and this requires setting a density/pressure floor for Hall-MHD to prevent negative pressure errors.
- The Hall-MHD implementation using the electron energy equation produces more negative pressure errors and requires higher floor values as compared to the Hall-MHD $\nabla p_e = \nabla p_i$ model.

More Conclusions

- The time step for Hall-MHD is restricted by the Whistler wave.
- The time step for the two-fluid model is restricted by the speed of light which is artificially lowered (as long as it is much larger than the Alfven speed). This requires less computational effort to resolve the two-fluid physics.
- Artificially reducing the ion-to-electron mass ratio reduces the computational effort of the two-fluid model in regimes where two-fluid physics is important.
- Close to the ideal-MHD regime, the two-fluid model become very stiff and Hall- and Ideal-MHD models are more computationally efficient.
- In the two-fluid regime, however, the two-fluid model provides comparable solutions to Hall-MHD while using less computational effort.

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