A Study of 3-Dimensional Plasma Configurations using the Two-Fluid Plasma Model

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Abstract & Motivation

- The two-fluid plasma model is studied and compared to reduced fluid models.
- The two-fluid model consists of the complete Euler equations for the ion and electron fluids and Maxwell’s equations for the electric and magnetic fields.
- Two-fluid effects become significant when the characteristic spatial scales are on the order of the ion skin depth and the characteristic time scales are on the order of the inverse ion cyclotron frequency. The Hall and diamagnetic drift terms capture the two-fluid physics.
Two-Fluid Plasma Model

Euler equations are used for ion and electron fluids denoted by subscript $s$.

\[
\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s u_s) = 0
\]

\[
\frac{\partial \rho_s u_s}{\partial t} + \nabla \cdot (\rho_s u_s u_s + \nabla p_s I) = \frac{\rho_s q_s}{m_s} (\mathbf{E} + u_s \times \mathbf{B})
\]

\[
\frac{\partial \epsilon_s}{\partial t} + \nabla \cdot ((\epsilon_s + p_s) u_s) = \frac{\rho_s q_s}{m_s} u_s \cdot \mathbf{E}
\]

\[
\epsilon_s \equiv \frac{p_s}{\gamma - 1} + \frac{1}{2} \rho_s u_s^2.
\]
Maxwell’s equations are used to evolve the electric and magnetic fields.

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0
\]

\[
\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = \mu_0 \sum_s \frac{q_s}{m_s} \rho_s u_s
\]

Perfectly hyperbolic Maxwell’s equations are used for divergence of \( \mathbf{B} \) and \( \mathbf{E} \) corrections

Dedner et al, Journal of Computational Physics, 2001
Hall-MHD Equation System by Applying Asymptotic Approximations

- Euler equations for ions and Faraday’s law same as two-fluid model.
- Quasi-neutrality assumption eliminates electron continuity.
- Neglecting electron inertia reduces electron momentum to Generalized Ohm’s law

\[ nq_e E = \nabla p_e - J_e \times B \]

- Infinite speed of light reduces Ampere’s law to

\[ J = \frac{1}{\mu_0} \nabla \times B \text{ where } J = J_i + J_e \]
Ideal-MHD Equation System

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 \\
\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u} + P \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{\mu_0} + \frac{B^2}{2\mu_0} \mathbf{I}) &= 0 \\
\frac{\partial \varepsilon}{\partial t} + \nabla \cdot \left[ \left( \varepsilon + P + \frac{B^2}{2\mu_0} \right) \mathbf{u} - \frac{(\mathbf{B} \cdot \mathbf{u})}{\mu_0} \mathbf{B} \right] &= 0 \\
\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{u} \mathbf{B} - \mathbf{B} \mathbf{u}) &= 0
\end{align*}
\]

The Ideal-MHD model does not contain the Hall and the diamagnetic drift terms that are present in Hall-MHD and the two-fluid model.
Computing Resources & Numerical Method

- WARPX (Washington Approximate Riemann Plasma) code
- Runge-Kutta, discontinuous-Galerkin (RKDG) method used
- Maui High Performance Computing Center’s Jaws system is used with 1000 CPU hours per simulation.
- ICE Cluster at the University of Washington is used
Axisymmetric Z-pinch Parameter Regime

\[
R_{\text{pinch}} = 2 \times 10^{-3} \, m
\]

\[
L_{\text{pinch}} = 2.5 \times 10^{-2} \, m
\]

\[
n_0 = 1 \times 10^{24} / m^3
\]

\[
T \approx 80 \, \text{eV}
\]

\[
l_0 = 40 \, kA
\]

\[
R_{\text{pinch}} = 8.6 \, R_{\text{Li}}
\]

\[
m_i = 25 \, m_e
\]

\[
V_{\text{light}} \approx 50 \, V_{\text{Alfven}}
\]
Axisymmetric Z-pinch Instabilities using Ideal-MHD

Evolution of Ideal MHD ion density from 0 to 4 Alfven transit times with a two wavelength perturbation.
Axisymmetric Z-pinch Instabilities using Two-Fluid

Evolution of Two-fluid ion density from 0 to 4 Alfven transit times with a two wavelength perturbation.
Axisymmetric Z-pinch Instabilities Growth Rates

- Growthrate of two-fluid and ideal MHD Z-pinch instabilities
- Fastest growing mode is the same for both for this regime
- Notice the symmetric evolution of the instability in ideal-MHD versus the sheared evolution in the two-fluid model
Axisymmetric Z-pinch Instabilities using Two-Fluid

Exploring a different parameter regime using normalized values, $R_{\text{pinch}} \approx 0.3 R_{\text{Li}}$, $\frac{m_i}{m_e} = 25$ and $V_{\text{light}} \approx 16 V_{\text{Alfven}}$

Left: Ideal-MHD evolution, Right: Two-fluid evolution after 1.25 Alfven transit times
Axisymmetric Z-pinch Instabilities Growth Rates

- At this different parameter regime initial growth rate same, two-fluid mode selection initially
- Two-fluid growth rate higher at later Alfven transit times
3-D Z-pincho using Two-Fluid Model

- Parameters: $R_{\text{pinch}} \approx \frac{1}{3} R_{Li}$, $\frac{m_i}{m_e} = 25$ and $V_{\text{light}} \approx 16 \ V_{\text{Alfven}}$
- Different from planar current sheet case because of radial pressure balance
- The presence of radial forces and azimuthal magnetic fields can lead to sausage and kink modes in the cylindrical configuration
- In addition to these, two-fluid instabilities are observed
3-D Z-pinch using Two-Fluid Model - Sausage mode

- Two-fluid model solution after $t = 0, 4t_A, 5t_A$
- Note small-wavelength lower hybrid drift instabilities on top of single wavelength perturbation
Two-fluid model solution after $t = 0, 4t_A, 5t_A$

Note small-wavelength lower hybrid drift instabilities on top of single wavelength perturbation
Axisymmetric Hill’s Vortex Parameter Regime

\[ R_s = 0.04 \text{m} \]
\[ R_c = 0.06 \text{m} \]
\[ L_s = 0.20 \text{m} \]
\[ n_0 = 2 \times 10^{22} / \text{m}^3 \]
\[ T \approx 100 \text{eV} \]
\[ s \approx 10(\text{kinetic parameter}) \]
\[ m_i = 25 \text{ } m_e \]
\[ V_{\text{light}} \approx 55 \text{ } V_{\text{Alfven}} \]
Axisymmetric Hill’s Vortex FRC Initial Condition

- Initial condition of the ion number density for an axisymmetric Hill’s vortex Field Reversed Configuration
- Initialization is for an MHD equilibrium
- Solved using the two-fluid plasma model
Axisymmetric Hill’s Vortex, Density, Toroidal field

- Left: Ion number density after 1.34 Alfven transit times
- Right: Toroidal magnetic field that develops in the plasma after 1.34 Alfven transit times
3-D Hill’s Vortex Initial Condition

Initial condition same as the axisymmetric case
Left: Cross-section of ion number density in \( x-z \) for midplane in \( y \)
Right: Cross-section of ion number density in \( x-y \) for midplane in \( z \)
Hill’s vortex FRC after 1.35 and 2.7 Alfvén transit times
Plots are of ion number density in \(x-y\) plane for midplane in \(z\)
No perturbation applied but note formation of small-wavelength instability
Conclusions

▶ Two-fluid model explored for 3-dimensional configurations of Z-pinch and a Hill’s vortex FRC.
▶ Small-wavelength instabilities seen in the axisymmetric and 3-D Z-pinch simulations
▶ Small-wavelength instabilities seen in the 3-D FRC that is not captured in the axisymmetric case
▶ Two-fluid model captures lower-hybrid drift instability in regimes where the ion Larmor radius is large
▶ Ideal-MHD works in small ion Larmor radius regimes
▶ Future Work: Examine if two-fluid model is stable to the $n = 0$ tilt instability in an FRC
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