Target Tracking Control with Limited Communication and Steering Dynamics

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***Abstract*— This paper presents a study which extends previous work on collective motion control of a group of vehicles for target tracking. The approach is to drive the group centroid to follow a moving target while keeping all vehicles near the centroid. The provided analysis shows that the system is stable. However, the tracking performance is sensitive to imperfect communication and steering dynamics. Simulation results show the performance degrades significantly when delays and steering dynamics are present. An analysis is provided to identify a major cause of the performance degradation. An alternative control law is proposed in the paper. Comparisons of the simulation results show the improved performance of the new control law.**

I. INTRODUCTION

Recently research focus in the control community has shifted from controlling machines like robotic manipulators to coordinating multi-agent systems. Research in multi-agent control can be applied to a number of practical engineering applications including the use of unmanned aerial vehicles in reconnaissance missions. A major difficulty in this shift of control paradigm is the limited communication among the agents. In the old paradigm concerning control of a single agent, the control law is designed with the assumption that the controller receives sensor information continuously or at a high update rate. In multi-agent systems, the assumption of continuous communication among the agents is not realistic. Virtually all communication devices have limited bandwidth and transmission delays. Agents can only communicate periodically, and message dropouts during the transmission commonly occur.

Work presented in this paper considers the problem of controlling a group of vehicles to collectively follow a moving target. Each vehicle's controller computes its own steering commands. A distributed control law is used to generate the steering commands. In the application domain considered here, the vehicles are assumed to be unmanned aerial vehicles following a ground target. For analysis and design purposes, the vehicle motion is described by a constant unit speed planar kinematic model. This unicycle model is used in many related works [1], [2]. Using complex notation, the kinematic model of vehicle k is given by

$$\frac{d}{dt}\begin{bmatrix} r_k \\ \theta_k \end{bmatrix} = \begin{bmatrix} e^{i\theta_k} \\ u_k \end{bmatrix}, \quad k \in \{1, 2, \ldots, N\}. \qquad (1)$$

The vector $r_k \in \mathbb{C}$ denotes the position of the vehicle in a 2-dimension inertial frame, and the angle $\theta_k \in \mathbb{S}^1$ denotes the orientation of the unit velocity vector $\dot{r}_k$. The vehicle is steered by the control input u_k. The index of the position vector r_k and orientation θ_k is omitted to represent the group position vector $r = [r_1, r_2, \ldots, r_N]$ and the group orientation vector $\theta = [\theta_1, \theta_2, \ldots, \theta_N]$ where N is the total number of vehicles.

Many groups of researchers use the above vehicle model to study collective motion control or formation control problems [3], [4], [5]. All of these works are based on the popular Kuramoto model of coupled oscillators [6]. Kim et al. [7] investigated the effects of limited communication to the stability of coupled oscillator systems. Another study of this problem can also be found in the work by Yeung and Strogatz [8]. Triplett et al. [9] presented a study of a discrete-time Kuramoto model which incorporates time delays. The study includes stability analyses of the system using linearization methods. There have been, however, few works which focus on coordinated target tracking problems subjected to limited communication. This paper presents a study of this topic.

The paper is organized as follows. Section II describes the control law used to compute steering commands for a group of vehicles to follow a target. The first two components of the control law are for matching the velocity and position of the group centroid to those of the target. The last term in the control law called spacing control is described in Section III. This section presents the result from a simulation with three vehicles to illustrate the effectiveness of the target tracking controller. Section IV presents a candidate cost function which is used to evaluate the performance of the controller. Sections V focuses on the study of the effects of limited communication and steering dynamics on the system performance. This section provides an analysis of the effects and presents an alternative spacing control law. Simulation results are provided to illustrate the performance of the controller with the new spacing control term and its robustness to communication delays and steering dynamics. The last section provides conclusions of this work and some future research directions.

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II. STEERING CONTROL

This section presents a solution to the target tracking control problem using a distributed control approach. The problem is to command a group of N vehicles to follow a moving target. Each vehicle is equipped with a controller which computes its own steering commands using sensor and communicated information. The control design presented here is based on the work by Klein and Morgansen [10]. The approach is to match the velocity and position of the group centroid to those of the target while keeping the vehicles near the centroid. Each vehicle has an identical controller. The group does not have a leader which performs special control actions.

The original design by Klein uses both feedback and feedforward techniques. The controller presented in this paper uses only feedback information to compute steering commands. The feedforward control is omitted because of both practical and theoretical reasons. Typically, in target tracking applications, the vehicles do not have the knowledge of future movement of the target. Each vehicle computes a steering command using the current position and velocity of the target provided by its onboard sensors. Klein's feedforward control law is formulated with the assumption that each vehicle has exact knowledge of the states of all other vehicles at all times. The assumption is not valid in this work where communication among the vehicles is limited. In addition, the performance of the feedforward control is susceptible to unmodelled dynamics of the vehicles. In this paper, changes in the target velocity are considered as disturbances to the control system.

The design scheme for the control problem is to decompose it into three subproblems: velocity matching, position matching, and maintaining relative spacing between each vehicle and the target. The steering control law for vehicle k takes the form

$$u_k = u_k^{match} + u_k^{track} + u_k^{space}. \quad (2)$$

The term u_k^{match} is for matching the velocity of the group centroid to the target velocity. The second term, u_k^{track}, is for reducing spatial errors between the centroid and the target. The last term, u_k^{space}, is for regulating the distance between each vehicle and the centroid. Assuming vehicles are unmanned aerial vehicles flying at different altitudes, collision avoidance is not considered in this control design.

The first subproblem is to match the velocity of the centroid $\dot{\bar{r}}$ to a reference velocity $\dot{r}_{ref}$, in this case, the velocity of the target $\dot{r}_t$. The velocity of the group centroid is a function of headings only:

$$\dot{\bar{r}} = \frac{1}{N}\sum_k e^{i\theta_k}. \quad (3)$$

Assuming the target velocity is constant and $\|\dot{r}_t\| < 1$, the control law in complex notation is given by

$$u_k^{match} = -K\langle \dot{\bar{r}} - \dot{r}_t, ie^{i\theta_k}\rangle, \quad K > 0 \quad (4)$$

The above function can be reformulated using sinusoidal terms and takes the form

$$u_k^{match} = K(-\frac{1}{N}\sum_j \sin(\theta_j - \theta_k) + v_t \sin(\theta_t - \theta_k)) \quad (5)$$

where v_t and θ_t are the speed and heading of the target, and K is a control gain. The control law in this form shows its similarity to the classic Kuramoto model of coupled oscillators [6]. The first term is the steering control function in the Kuramoto model which depends only on relative headings. Since $K > 0$, this term drives the group linear momentum to zero [11]. The second term is the control which guides the group centroid to follow the target. This control law is based on a gradient control method which defines the control function as

$$u_k^{match} = -K\frac{\partial V}{\partial \theta_k} \quad (6)$$

where V is a Lyapunov function given by

$$V = \frac{1}{2}N\|e_v\|^2. \quad (7)$$

The stability analysis by Klein [10] using Lasalle's Invariance Principle shows that the control law asymptotically stabilizes the error between the velocity of the centroid and the target velocity,

$$e_v = \dot{\bar{r}} - \dot{r}_t. \quad (8)$$

A system starting at any initial state outside the invariant set

$$E = \{\theta | \langle \dot{\bar{r}}(\theta) - \dot{r}_t, ie^{i\theta_k}\rangle = 0, \forall k\}. \quad (9)$$

will converge to a stable equilibrium point within the set. The set E, however, also contains unstable equilibria which are described by the condition $\langle \dot{\bar{r}}(\theta) - \dot{r}_t, ie^{i\theta_k}\rangle = 0$ and $\dot{\bar{r}}(\theta) \neq \dot{r}_t$.

The second subproblem is to regulate the spatial error between the centroid and the target. This goal can be achieved by manipulating the reference velocity $\dot{r}_{ref}$. The concept is to adjust the direction and magnitude of the reference velocity such that the velocity vector points toward the target when there exists an error. The reference velocity must match the target velocity when there is no spatial error. This reference velocity can take the form

$$\dot{r}_{ref} = (1-w)\dot{r}_t + w\frac{r_t - \bar{r}}{\|\bar{r} - r_t\|} \quad (10)$$

where w is a weighting function given by

$$w = 1 - e^{-\alpha\|\bar{r} - r_t\|} \quad (11)$$

and α is a constant. The condition

$$\lim_{\|\bar{r} - r_t\| \to 0} \frac{r_t - \bar{r}}{\|\bar{r} - r_t\|} = 0 \quad (12)$$

is required to meet the concept described above.

The control law for matching both velocity and position of the centroid to the target can be written as

$$u_k = -K\langle \dot{\bar{r}} - \dot{r}_{ref}, ie^{i\theta_k}\rangle, \quad K > 0. \quad (13)$$

Using (10), the control law becomes

$$u_k = -K\langle\dot{\bar{r}}-\dot{r}_t, ie^{i\theta_k}\rangle - K\langle w\dot{r}_t - w\frac{r_t-\bar{r}}{\|\bar{r}-r_t\|}, ie^{i\theta_k}\rangle. \quad (14)$$

The first term is the velocity matching term defined in (4). The second term is the position matching control

$$u_k^{track} = -K\langle w\dot{r}_t - w\frac{\bar{r}-r_t}{\|\bar{r}-r_t\|}, ie^{i\theta_k}\rangle. \quad (15)$$

This control function, introduced in this paper, can be written in a sinusoidal form as

$$u_k^{track} = -K\left(wv_t\sin(\theta_t-\theta_k) - w\sin(\phi_{ct}-\theta_k)\right) \quad (16)$$

where ϕ_{ct} is the angle of the position vector from the centroid to the target location relative to the x axis of the inertial coordinate frame.

Theorem 1: Target tracking control. The steering control defined in (14) with any smooth weighting function $0 < w < 1$ and the condition defined in (12) will drive the position of the centroid to converge to the target position assuming the error between the velocity of the centroid and the reference velocity is smaller than $w(1-\|\dot{r}_t\|)$.

Proof: The proof follows the Lyapunov second theorem. Given a candidate Lyapunov function:

$$V = \frac{1}{2}\|\bar{r}-r_t\|^2, \quad (17)$$

the time derivative of V can be written as

$$\dot{V} = \langle\bar{r}-r_t, \dot{\bar{r}}-\dot{r}_t\rangle. \quad (18)$$

The velocity error is defined as

$$\dot{\varepsilon} = \dot{\bar{r}} - \dot{r}_{ref}. \quad (19)$$

Using the above definition, (18) becomes

$$\dot{V} = \langle\bar{r}-r_t, \dot{r}_{ref}-\dot{r}_t+\dot{\varepsilon}\rangle. \quad (20)$$

Using (10), this equation can be rewritten as

$$\begin{aligned}\dot{V} &= \left\langle \bar{r}-r_t, (1-w)\dot{r}_t - w\frac{(\bar{r}-r_t)}{\|\bar{r}-r_t\|} - \dot{r}_t + \dot{\varepsilon}\right\rangle \\ &= \left\langle \bar{r}-r_t, -w\dot{r}_t - w\frac{(\bar{r}-r_t)}{\|\bar{r}-r_t\|} + \dot{\varepsilon}\right\rangle \\ &= -\langle\bar{r}-r_t, w\dot{r}_t\rangle - w\|\bar{r}-r_t\| + \langle\bar{r}-r_t, \dot{\varepsilon}\rangle.\end{aligned}$$

Proofing by contradiction, we assume that $\dot{V} \geq 0$. Using the assumption, it can be concluded from the above equation that

$$\frac{\langle\bar{r}-r_t, \dot{r}_t-(\dot{\varepsilon}/w)\rangle}{\|\bar{r}-r_t\|} \leq -1. \quad (21)$$

The worst-case lower bound of the left-hand side term is $-(\|\dot{r}_t\| + \|\dot{\varepsilon}/w\|)$, so the condition in (21) fails when

$$\|\dot{r}_t\| + \|\dot{\varepsilon}/w\| < 1. \quad (22)$$

As a result, the time derivative of the Lyapunov function is negative definite given the stability condition

$$\|\dot{\varepsilon}\| < w(1-\|\dot{r}_t\|) \quad (23)$$

which requires that $\|\dot{r}_t\| < 1$ and w must be positive. ∎

III. SPACING CONTROL

This section describes the last term in the steering control law defined in (2) which is called spacing control. This term is added to keep individual vehicles near the centroid. Without the spacing term, the control law given in (14) would drive individual vehicles to diverge from the centroid although their centroid is converging to the target. One solution suggested by Klein is to add a spacing control term which satisfies the condition

$$\frac{1}{N}\sum_k ie^{i\theta_k}u_k^{space} = J\mathbf{u}^{space} = 0 \quad (24)$$

where $\mathbf{u}^{space} = [u_1^{space}, u_2^{space}, \ldots, u_N^{space}]^T$ and the Jacobian matrix J is given by

$$J = \frac{i}{N}\left[e^{i\theta_1}, e^{i\theta_2}, \ldots, e^{i\theta_N}\right]. \quad (25)$$

This spacing control does not affect the convergence property of the centroid matching control law. Any spacing control vector which is in the kernel space of the Jacobian matrix

$$\mathbf{u}^{space} \in \ker\{J\} \quad (26)$$

will satisfy the condition in (24). The spacing control is in a $N-2$ dimensional space because two degrees of freedom are needed by velocity matching constraints,

$$\dot{\bar{r}} = \frac{1}{N}\sum_k e^{i\theta_k} = \dot{r}_{ref}. \quad (27)$$

Thus, this spacing control can only be used for a group of more than two vehicles i.e. $N > 2$. Also note that a spacing control law which satisfies the condition in (24) may not necessarily keep the vehicles near the centroid. Finding a spacing control law for N vehicles, which satisfies the condition while keeping the vehicles near the centroid, is not trivial. However for three vehicles, Klein introduced an admissible control law which is given by

$$\begin{bmatrix} u_1^{space} \\ u_2^{space} \\ u_3^{space} \end{bmatrix} = \begin{bmatrix} \sin(\theta_3-\theta_2) \\ \sin(\theta_1-\theta_3) \\ \sin(\theta_2-\theta_1) \end{bmatrix} \quad (28)$$

Fig. 1 shows the result from a simulation of three vehicles tracking a target using the control functions defined in (4), (15), and (28). The control parameters used in the simulation are $\alpha = 0.3$ and $K = 0.5$. In the figure, the dashed red line represents the target trajectory during the mission. The black line is the trajectory of the group centroid. The blue, green, and cyan lines represent the trajectories of the three vehicles. In this simulation, the target started at coordinate (5,5) and headed east with a constant speed of 0.3 (units are irrelevant, so are omitted here). The target turned left at the rate of 0.05 rad/s during the time 100 to 150 second and maintained its course afterward. All three vehicles had constant unit speed and were able to communicate with one another at all times. The vehicles are assumed to be equipped with sensors which can provide the true position and velocity

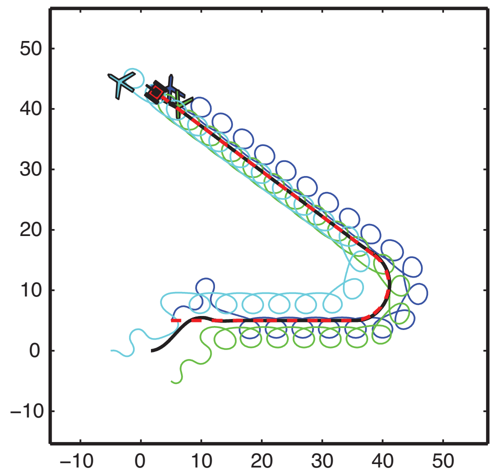


Fig. 1. Trajectories of three vehicles following a target assuming continuous communication and no steering dynamics.

of the target. The simulation result shows the vehicles were able to follow the target effectively even during the unexpected turn of the target. The centroid converges to the target quickly, and the controller can adjust to the unexpected turn. This simulation result is similar to the result presented in Klein's work even though the feedforward control term is not included here.

IV. PERFORMANCE EVALUATION

Performance of a control system can be quantified using a cost function. The best controller would yield the minimum value of the cost function. This cost function should capture the design objectives and include the required specifications of the controller. The main objective of the control design presented in the previous sections is to command a group of vehicles to follow a moving target. The approach is to keep both the group centroid and individual vehicles near the target. A candidate cost function which reflects the design objectives can take the form

$$J = \sqrt{\frac{1}{N_T}\sum_{T=0}^{N_T}\rho_{ct}^2} + \sum_{k=1}^{N}\sqrt{\frac{1}{N_T}\sum_{T=0}^{N_T}h^2(\rho_{kt})} \tag{29}$$

where N_T is the total number of time steps during the mission, ρ_{ct} is the distance from the centroid to the target, and ρ_{kt} is the distance from vehicle k to the target. The function $h(\rho_{kt})$ is given by

$$h(\rho_{kt}) = \begin{cases} (\rho_{kt} - \rho_{t0}) & , \rho_{kt} \geq \rho_{t0} \\ 0 & , \rho_{kt} < \rho_{t0} \end{cases} \tag{30}$$

Here, ρ_{t0} is the reference tracking distance. The first term in the cost function is the root mean square of the centroid spatial errors. The second term penalizes the distance from each vehicle away from the target if it is larger than the radius of the target area ρ_{t0}. This cost function does not include a term which penalizes the closeness between each pair of the vehicles because collision avoidance is not a concern in this research. This cost function is used in the next section to evaluate the performance of the controller.

V. EFFECTS OF DELAYS AND STEERING DYNAMICS

In real-world applications, the assumption that vehicles can continuously communicate is not valid. The communication among vehicles usually occurs intermittently and the transmission of the messages is typically delayed. Steering dynamics is another practical issue that should be considered in the controller design. Vehicles' actuators cannot instantaneously respond to the commanded turn rates. The controller design and simulation result presented in Section II and III do not take into account the effects of imperfect communication and steering dynamics. The main focus in this section is to study these effects on the controller's performance. The following assumptions are used in this study.

- The communication network is fully connected.
- Each vehicle can only send messages periodically.
- Each vehicle broadcasts its current state to all other vehicles.
- All transmissions of messages have the same constant delay time.

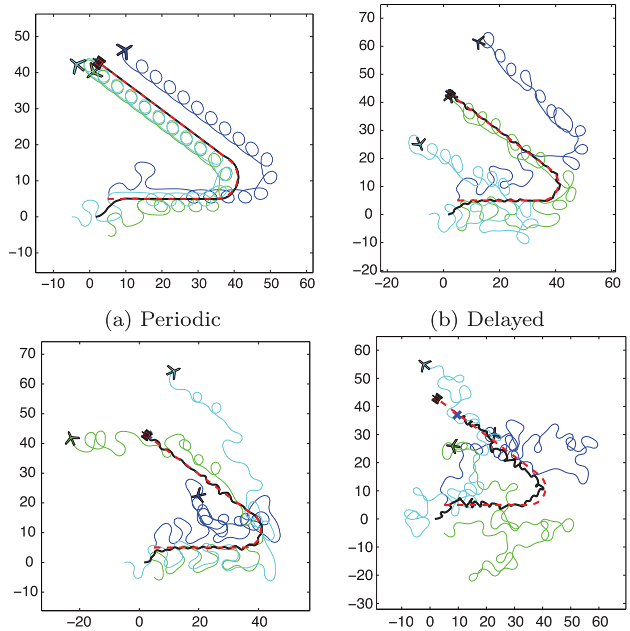
The last assumption means that, for any pair of vehicles (i, j), the state information of vehicle i, $x_i(t)$, transmitted at time t will be received by vehicle j at time $t+d$ where d is the constant delay time.

By adding first-order steering dynamics to the original model given in (1), the new vehicle model is in the form

$$\frac{d}{dt}\begin{bmatrix} r_k \\ \theta_k \\ \omega_k \end{bmatrix} = \begin{bmatrix} e^{i\theta_k} \\ \omega_k \\ (-\omega_k + u_k)/\tau_\omega \end{bmatrix}. \tag{31}$$

The added state variable ω_k is the turn rate of vehicle k, and τ_ω is the time constant of the steering dynamics.

Simulations were performed to assess the performance of the controller in four different configurations. The configurations were chosen to show the effects of delays and steering dynamics. In the simulations, each vehicle broadcasts its state information to all other vehicles every second. Each vehicle has an onboard sensor which provides the true state of the target with an update rate of 1 second. The first simulation configuration has the same set-up as the simulation presented in Section III except the periodic communication model. Transmission delays are added to the second configuration. In the third configuration, the vehicle model with steering dynamics ($\tau_\omega = 1$) replaces the original unicycle model. The last configuration includes both the delays and steering dynamics. The simulation results of all four configurations are shown in Figs. 2-(a), (b), (c), (d) respectively. Fig. 3 shows the the comparison of the spatial errors between the centroid and the target position. Fig. 4 shows the comparison of the cost during the mission. The cost function, defined in (29), represents how well the vehicles follow the target.



(a) Periodic (b) Delayed

(c) Steering Dynamics (d) Combined

Fig. 2. Simulation results of the four configurations. The plots show trajectories of the vehicles and the target during the mission.

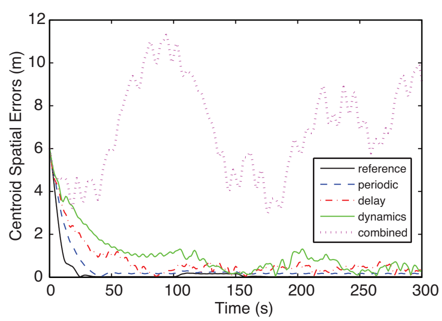


Fig. 3. Comparison of the spatial errors between the centroid and the target. The result from the continuous case is shown as a based-line reference.

[htb]

The simulation results show that the controller performed best in the ideal case which had perfect communication and no steering dynamics. The tracking performance slightly degraded when the vehicles can only communicate periodically every one second. The vehicles and their centroid still effectively followed the target closely at all times during the mission. The performance of the controller significantly degraded when either communication delays or steering dynamics was included in the simulation. In both cases, the vehicles stayed further away from the target. The degraded performance caused the increase in mission cost. In the last configuration where both delays and steering dynamics were included,

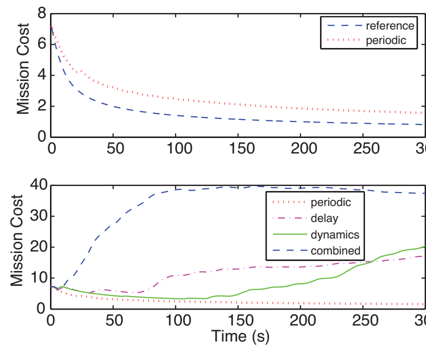


Fig. 4. Comparison of the cost during the mission with $\rho_{t0} = 10$. The result from the continuous case is shown as a based-line reference.

the controller had the worst performance. The mission cost increased greatly as shown in Fig. 4. Fig. 3 illustrates that the group centroid did not converge to the target. These results show that the controller is not robust to communication delays and steering dynamics. The combined effects considerably degrade the performance. This pattern of performance degradation also appears in the other simulations using different control gains and scenario variables.

The communication delays and steering dynamics affect both the centroid matching and spacing control components. However, the performance degradation, seen in the simulation results, is mainly caused by the spacing control component. As mentioned in Section III, the spacing control term is added to keep individual vehicles near the centroid. The approach is to add a periodic function to the control law such that the added term collectively does not affect the centroid velocity. This feedback control law is designed with the concept of feedforward control which compensates for errors directly assuming the controller has accurate system parameters. The problem is that this control is sensitive to these parameters which, in this case, are the headings of neighbor vehicles. In the ideal situation with continuous communication and no steering dynamics, each vehicle has accurate states of all other vehicles. Thus, the controller performs well using the spacing control. When vehicles can only communicate periodically and delays are introduced to the system, the controller cannot compute the spacing control accurately. That limitation results in degraded performance.

Adding steering dynamics invalidates the proposition that any spacing control vector in the kernel space of the Jacobian matrix will not affect the centroid velocity. In the ideal unicycle model, the centroid acceleration is given by

$$\ddot{r} = \frac{1}{N}\sum_k ie^{i\theta_k}\dot{\theta}_k = \frac{1}{N}\sum_k ie^{i\theta_k}u_k. \tag{32}$$

Thus, any spacing control which satisfies

$$\frac{1}{N}\sum_{k} ie^{i\theta_k}u_k^{space} = J\mathbf{u}^{space} = 0 \tag{33}$$

will not change the centroid velocity. However, equation (32) does not hold when steering dynamics are introduced to the system. As a result, the added dynamics invalidate both the condition in (33) and the stability proof of the control law.

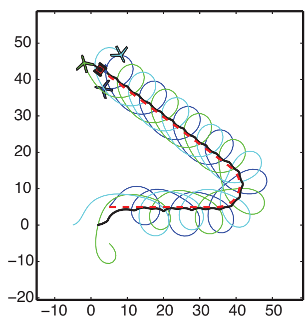
One possible solution to this problem is to replace the original spacing control with the beacon control introduced by Paley et al. [12]. The concept is to control each vehicle to circle around a beacon at a fixed radius ρ_0. The control law takes the form

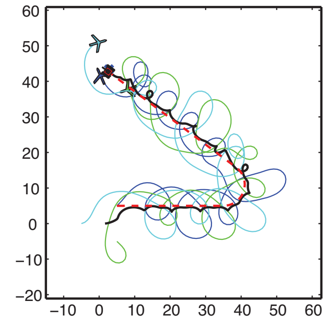
$$u_k^{space} = -\omega_0 - \omega_0 K_s\langle r_k - r_t, \dot{r}_k\rangle \tag{34}$$

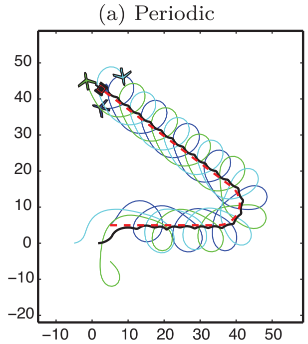
where K_s is a control gain. The first term in the control law drives the vehicle to circle around the target in the clockwise direction with radius $\rho_0 = 1/\omega_0$. The dissipation in the second term is added to stabilize the circular motion about the beacon which is the target in this case. The main advantage of the beacon control is that the control law does not need information of other vehicles' states. It is only a function of the vehicle's own state and the position of the target. Communication limitations do not affect the beacon control law, but they still have effects on the performance of the centroid matching control. Furthermore, the beacon control law can be applied to any number of vehicles whereas the spacing control law proposed by Klein is limited only for a group of three vehicles.

When the target is stationary, using the beacon control law with the centroid matching control defined in section II, the system will converge to the "splay state" where all vehicles are tracing the same circular orbit around the fixed target position. The vehicles are evenly distributed around the target and have equal phase shifts relative to their neighbors. In this case, the control law presented in this paper is equivalent to the one proposed by Paley et al. as described in [12]. They showed that this control law is asymptotically stable in Lyapunov sense. The motion of the target can be considered as a perturbation to the splay state. The question is how well the controller compensates for this perturbation given limited communication and the presence of steering dynamics.

The same four simulation configurations previously used for testing the original control law are applied here to evaluate the new control law. The beacon control parameters used in the simulations are $\omega_0 = 0.2$ and $K_s = 0.1$. Trajectories of the vehicles and the target from all configurations are illustrated in Fig. 5. Fig. 6 shows the the comparison of the spatial errors between the centroid and the target position. Fig. 7 shows the comparison of the cost during the mission from the four simulation configurations. Figure 8 shows the comparison of the overall cost values obtained from two sets of simulations using the original control law and the new


(a) Periodic


(b) Delayed


(c) Steering Dynamics

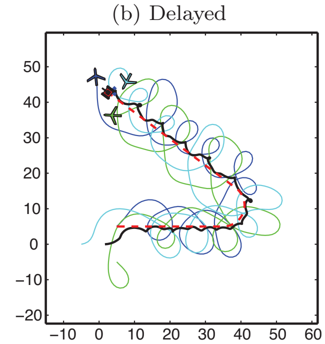

(d) Combined

Fig. 5. Simulation results of the four configurations using beacon control. The plots show trajectories of the vehicles and the target during the mission.

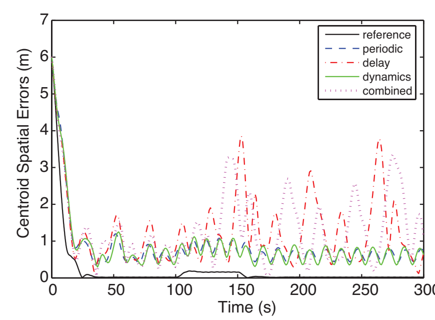


Fig. 6. Comparison of the spatial errors between the centroid and the target. The result from the continuous case is shown as a based-line reference.

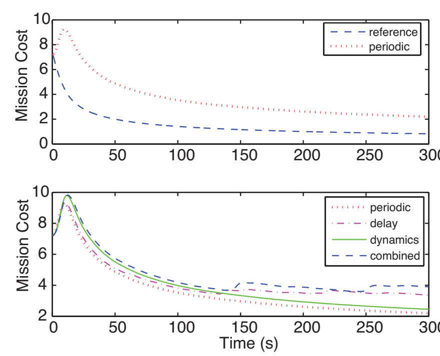


Fig. 7. Comparison of the cost during the mission with $\rho_{t0} = 10$. The result from the continuous case is shown as a based-line reference.

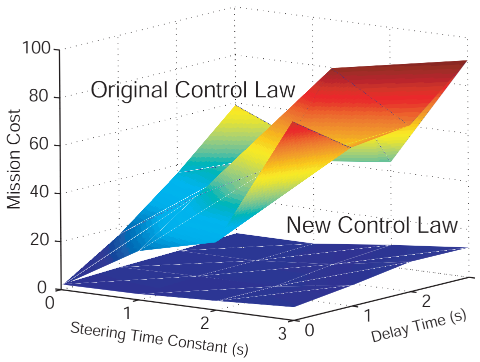


Fig. 8. Comparison of the overall costs with different delay times ΔT and steering time constants τ_ω.

control law. The results were obtained by varying the delay time and the time constant in the steering dynamic model.

Simulation results show that the performance of the new controller was not much affected by delays and steering dynamics compared to the original one. In the first simulation configuration, the vehicles were able to effectively follow and orbited around the target. The overall cost slightly increased compared to the ideal case with continuous communication. Adding delays and/or steering dynamics increased the mission cost, but the increase was significantly smaller than that in the case when the original control law was applied. In all configurations, the vehicles, as well as their centroid, followed the target closely at all times.

VI. CONCLUSIONS

This paper presents a study of target tracking control problems subjected to the effects of limited communication and steering dynamics. The study focuses on the approach proposed by Klein and Morgansen [10]. The approach is applied to control a group of three vehicles to follow a moving target. Simulation results show that the vehicles were able to follow the target effectively, but the system performance was sensitive to communication delays and steering dynamics. The combined effects caused the system to become ineffective. The provided analysis shows that a major cause of the performance degradation is in the original design of the spacing control. Replacing the original spacing control law with the beacon control function improves the system performance. Simulation results show that the new controller performed effectively even when both communication delays and steering dynamics were presented.

Further investigation can be done in many other aspects related to this work. In this paper, the control law formulation and stability analysis are conducted in the continuous time domain. To study the effects of intermittent and randomly delayed communication, it is more natural to formulate the problem and design the control law in the discrete time domain. Another area for future work is the study of the effects of different communication network topologies and random message dropouts.

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