THE SHEARED-FLOW STABILIZED Z-PINCH

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The stabilizing effect of a sheared axial flow is investigated in the ZaP Flow Z-pinch experiment at the University of Washington. Long-lived, Z-pinch plasmas are generated that are 100 cm long with a 1 cm radius and exhibit gross stability for many Alfvén transit times. Experimental measurements show a sheared flow profile that is coincident with the quiescent period during which magnetic fluctuations are diminished. The flow shear is generated with flow speeds less than the Alfvén speed. While the electrodes contact the ends of the Z-pinch, the surrounding wall is far enough from the plasma that the wall does not affect stability, as is investigated experimentally and computationally. Relations are derived for scaling the plasma to high energy density and to a fusion reactor. The sheared flow stabilized Z-pinch concept provides a compact linear system.

I. INTRODUCTION

Nuclear fusion holds the promise of a high-energy-density power source with plentiful fuel. However, plasma stability has complicated the search for a viable reactor configuration for controlled fusion. In addition to a sufficiently high temperature, the product of the plasma density and confinement time must exceed a value called the Lawson criterion for breakeven conditions. Generally, magnetic confinement fusion aims to confine a low-density plasma for a long time, and inertial confinement fusion aims to confine a high-density plasma for a short time. Plasma stability must be addressed for both magnetic confinement and inertial confinement.

Magnetic confinement often addresses plasma stability with the addition of magnetic fields, whose primary purpose is not to confine the plasma but to provide stability. The classical Z-pinch configuration has no stabilizing magnetic fields. It is a simple cylindrical equilibrium described by the radial force balance between the plasma pressure p and the azimuthal magnetic field B_θ given by

$$\frac{B_\theta}{\mu_0 r}\frac{d}{dr}\left(rB_\theta\right) = -\frac{d}{dr}\left(p\right). \tag{1}$$

This equilibrium has unit average plasma β (ratio of plasma pressure to magnetic pressure). The Z-pinch equilibrium is unstable to modes that grow exponentially in time, but these modes can be stabilized with the addition of an axial magnetic field B_z. The equilibrium is then given by

$$\frac{B_\theta}{\mu_0 r}\frac{d}{dr}\left(rB_\theta\right) = -\frac{d}{dr}\left(p + \frac{B_z^2}{2\mu_0}\right). \tag{2}$$

The required axial magnetic field is given by the Kruskal-Shafranov limit. Adding the axial magnetic field decreases the plasma β and increases the complexity and expense of a fusion device. However, the Z-pinch can also be stabilized by driving a sheared flow in the plasma.

I.A. Sheared-Flow Stabilized Z-Pinch

The sheared-flow stabilized Z-pinch uses radial shear of the axial plasma velocity, dv_z/dr, to provide stability. No axial magnetic fields are used to provide stability; only the sheared flow provides stability. The equilibrium given by Eq. (1) is unaffected by an axial plasma flow. Therefore, it still has unit average plasma β. The sheared-flow stabilization has been investigated experimentally in the ZaP Flow Z-pinch. The experimental operation and the sheared flow stabilization results are described in detail in Refs. 1-4. A drawing of the experiment is shown in Fig. 1 identifying the major components, such as the accelerator and pinch assembly regions.

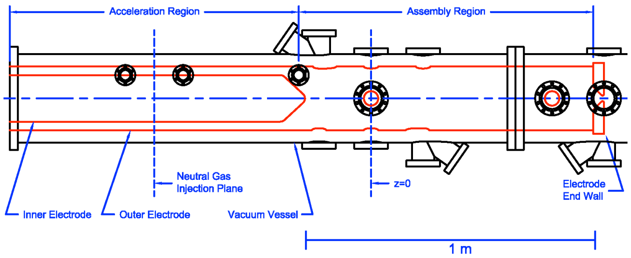

Fig. 1. Machine drawing of the ZaP experiment identifying the major components. A 1 m scale is placed for reference.

The flow Z-pinch plasma is formed by injecting neutral gas, e.g. hydrogen or helium, into the annulus between the inner and outer electrodes. An electrical potential is applied across the electrodes, breaking down the gas, forming the plasma, and accelerating it into the assembly region. The plasma converges onto the axis in the pinch assembly region forming a 1 m long flow Z-pinch. Inertia maintains the axial plasma flow. Remnant currents in the accelerator continuously replenish the plasma as it exits through the exhaust hole in the electrode end wall.

During the initial pinch assembly, multiple diagnostics measure large plasma fluctuations. After the pinch assembles, an extended quiescent period is observed where the plasma fluctuations are low and the plasma is centered and uniform in the assembly region. The quiescent period is defined by plasma fluctuations that are below those corresponding to a plasma displacement equal to a plasma radius. The quiescent period is coincident with a measured sheared flow. After the quiescent period, large plasma fluctuations are again measured.

II. NO WALL STABILIZATION

A conducting wall can stabilize the pure Z-pinch by providing local image currents to maintain the axisymmetric and axial uniformity; however, the wall has to be located close to the plasma edge, $r_{wall}/a < 1.2$ (Ref. 5), which is incompatible with a high temperature plasma. If the wall is positioned beyond $r_{wall}/a = 4$, then the wall provides no stabilization as demonstrated in Ref. 5.

In the ZaP experiment, the plasma radius a is approximately 1 cm giving a value of $r_{wall}/a \approx 10$ for the experimental geometry, which should eliminate any wall stabilization effect. The wall stabilization effect is further investigated in the ZaP experiment by perforating the outer electrode with large gaps, such that the outer electrode has a section comprised of eight axial bars. The perforated outer electrode is shown in Fig. 2. Note that most of the outer electrode material has been removed, reducing azimuthal image currents and allowing the plasma to escape.

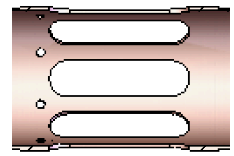

Fig. 2. Perforated outer electrode extension used to experimentally investigate wall stabilization in ZaP.

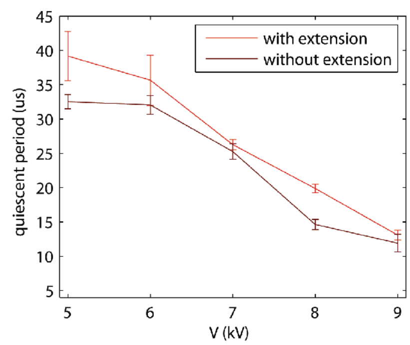

Fig. 3. Experimental results showing the variation of the quiescent period length with applied potential with and without the perforated outer electrode extension. The error bars indicate the pulse-to-pulse standard deviation for 10 – 16 pulses.

The experimental results show that plasma stability is not affected by the perforations in the outer electrode. Specifically, the axisymmetric and axial uniformity of the plasma structure is maintained throughout the quiescent period. Figure 3 shows the variation of the quiescent period length both with and without the perforated outer electrode extension.

The wall stabilization effect is also investigated computationally with a nonlinear resistive MHD code, HiFi, which is a three-dimensional, multi-block extension of the SEL code[6]. The growth of the kinetic energy, corresponding to the growth of an unstable mode, is shown in Fig. 4. The simulations with a solid axisymmetric conducting wall generally agree with the wall stabilization effect reported in Ref. 5. Since HiFi uses a multi-block grid, it is capable of simulating complicated geometries[7]. The perforated outer electrode is simulated by extending the simulation domain radially through the large gaps in the outer electrode. The computational domain with the initial condition is shown in Fig. 5. The initially-static equilibrium develops a kink mode structure later in time as seen in Fig. 5. The growth of the kinetic energy for the plasma surrounded by the perforated outer electrode is shown in Fig. 4. The growth rate is the same as the case without perforations.

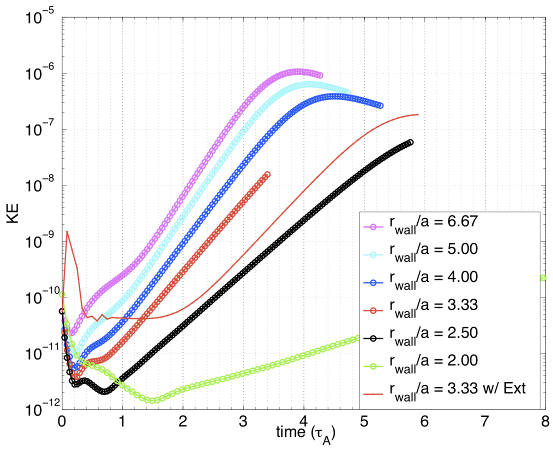

Fig. 4. Computational results showing the evolution of the kinetic energy for different wall locations. Traces with circular symbols represent simulations with an axisymmetric outer electrode. The trace without symbols represents the simulation with the perforated outer electrode.

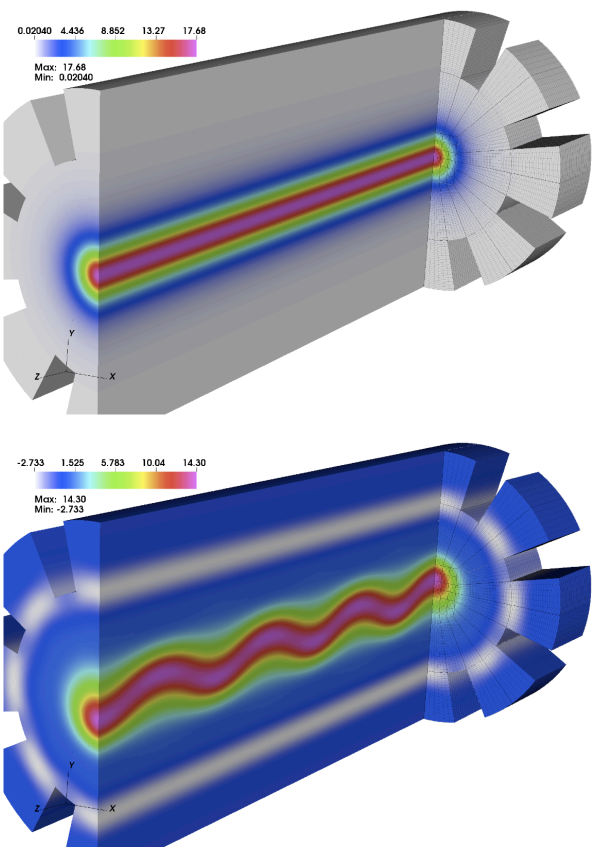

Fig. 5. Three-dimensional simulation results of an initially-static Z-pinch equilibrium in the perforated outer electrode geometry to investigate the effect of wall stabilization.

In both the experimental and computational investigations plasma stability is unaffected by the perforations in the outer electrodes. Therefore, the results indicate that no wall stabilization is present. The conducting wall can be placed arbitrarily far from the plasma volume.

III. SCALING RELATIONS

The integral force balance gives the Bennett pinch relation

$$\left(1+Z\right)NkT = \frac{\mu_0 I^2}{8\pi}, \tag{3}$$

where N is the linear density defined as

$$N = \int_0^a n(r) 2\pi r dr. \tag{4}$$

The Bennett relation is valid for any equilibrium profile. From this relation alone, the temperature scaling is given as

$$\frac{T_2}{T_1} = \left(\frac{I_2}{I_1}\right)^2 \frac{N_1}{N_2}. \tag{5}$$

If the equilibrium is modified such that the plasma responds adiabatically, then the changes in pressure and density are related by

$$0 = \frac{d}{dt}\left(\frac{p}{n^\gamma}\right) = \frac{d}{dt}\left(\frac{\left(1+Z\right)kT}{n^{\gamma-1}}\right). \tag{6}$$

The density then scales as

$$\begin{aligned} \frac{n_2}{n_1} &= \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}} \\ &= \left(\frac{I_2}{I_1}\right)^{\frac{2}{\gamma-1}} \left(\frac{N_1}{N_2}\right)^{\frac{1}{\gamma-1}}. \end{aligned} \tag{7}$$

An equilibrium is assumed with a uniform density, $n_i = n$, $n_e = Z\ n$, and temperature $T_i = T_e = T$ within the characteristic pinch radius a and both are zero for all radii greater than a. This provides expressions for the peak magnetic field and the density

$$p = (1+Z)nkT = \frac{B^2}{2\mu_0},$$
$$B = \frac{\mu_0 I}{2\pi a}, \qquad (8)$$
$$n = \frac{N}{\pi a^2}.$$

Combining these expressions with Eqs. (5) and (7), scaling relations for the pinch radius, magnetic field, and pressure can be found as

$$\frac{a_2}{a_1} = \sqrt{\frac{n_1}{n_2}\frac{N_2}{N_1}} = \left(\frac{I_1}{I_2}\right)^{\frac{1}{\gamma-1}}\left(\frac{N_2}{N_1}\right)^{\frac{\gamma}{2(\gamma-1)}},$$
$$\frac{B_2}{B_1} = \frac{a_1}{a_2}\frac{I_2}{I_1} = \left(\frac{I_2}{I_1}\right)^{\frac{\gamma}{\gamma-1}}\left(\frac{N_1}{N_2}\right)^{\frac{\gamma}{2(\gamma-1)}}, \qquad (9)$$
$$\frac{p_2}{p_1} = \frac{n_2}{n_1}\frac{T_2}{T_1} = \left(\frac{I_2}{I_1}\right)^{\frac{2\gamma}{\gamma-1}}\left(\frac{N_1}{N_2}\right)^{\frac{\gamma}{\gamma-1}}.$$

III.A. Scaling to High Energy Density Plasma

The scaling relations given by Eqs. (5), (7), and (9) provide a path to achieve high energy density plasmas (HEDP). As an example, starting with a plasma described by I_p = 50 kA, T = 20 eV, a = 10 mm, and $n = 6\times10^{22}$ m^{-3}, increasing the current yields the final plasma parameters shown in Fig. 6. A point design is shown for a plasma current of 750 kA which produces an HEDP of $p = 3\times10^{11}$ Pa, T = 4.5 keV, a = 0.17 mm, and $n = 2\times10^{26}$ m^{-3}, assuming the linear density N remains constant.

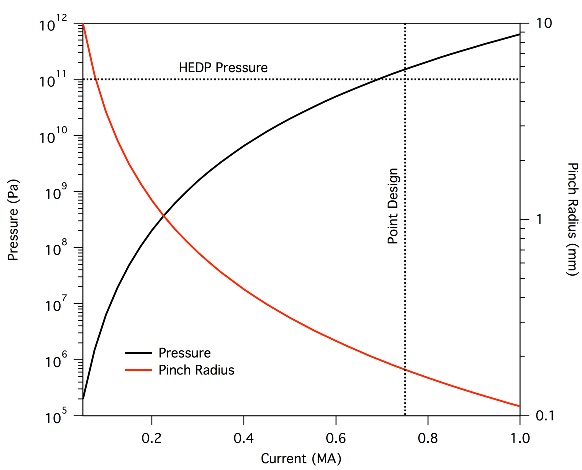

Fig. 6. Plasma parameters scaled from an initial plasma described by I_p = 50 kA, T = 20 eV, a = 10 mm, and $n = 6\times10^{22}$ m^{-3}. The typical definition of HEDP uses a pressure of 10^{11} Pa.

III.B. Scaling to a Fusion Reactor

A fusion reactor can be designed based on the sheared flow-stabilized Z-pinch. For a D-T reaction, the fusion power production is computed as

$$P_f = n_D n_T \langle\sigma v\rangle_{DT} E_{DT}\pi a^2 L = \frac{n^2}{4}\langle\sigma v\rangle_{DT} E_{DT}\pi a^2 L, \qquad (10)$$

where the densities are defined as $n = n_D + n_T$ and a 50-50 mixture of D-T, $n_D = n_T$ is assumed. The energy released, E_{DT}, for each D-T fusion event is 17.6 MeV. The reaction rate parameter in units of m^3/s can be expressed as

$$\langle\sigma v\rangle_{DT} = 3.68\times10^{-18} T_{\mathrm{keV}}^{-\frac{2}{3}} \exp\left(-19.94 T_{\mathrm{keV}}^{-\frac{1}{3}}\right). \qquad (11)$$

The input power for the flow-stabilized Z-pinch includes the thermal power, flow power, and radiative power, $P_{in} = P_{th} + P_{flow} + P_{rad}$. The flow and thermal powers are losses. The thermal power is the power required to heat and compress the reactants to fusion conditions and is given by

$$P_{th} = \frac{3}{2}\sum_\alpha \dot{n}_\alpha k T_\alpha \pi a^2 L \quad \text{for } \alpha = D,T,e,e$$
$$= 3\dot{n}kT\pi a^2 L \qquad (12)$$
$$= 3nkTv_z\pi a^2,$$

where $\dot{n} = nv_z/L = n/\tau_{flow}$. The plasma and its energy are assumed to be confined for the duration of the flow-through time, τ_{flow}. The flow-stabilized Z-pinch requires power, P_{flow}, to drive the axial flow of the plasma, which is given by

$$P_{flow} = \frac{1}{2}\dot{m}v_z^2$$
$$= \frac{M_D + M_T}{4}\dot{n}v_z^2\pi a^2 L \qquad (13)$$
$$= \frac{M_D + M_T}{4}nv_z^3\pi a^2.$$

Stability requires an axial flow speed greater than a fraction of the Alfvén speed such that $v_z \geq 0.1\ v_A$ for a uniform shear and $ka = 1$.

The radiative power loss for a D-T plasma is primarily due to bremsstrahlung radiation, which is given by

$$P_{rad} = C n_e T_e^{\frac{1}{2}} \sum_i Z_i^2 n_i$$
$$= 3.4 \times 10^{-38} n_{\mathrm{m}^{-3}}^2 T_{\mathrm{eV}}^{\frac{1}{2}}, \tag{14}$$

since $n = n_D + n_T = n_e/2$.

Fusion gain, Q, is defined as the ratio of fusion power to input power and is expressed as

$$Q \equiv \frac{P_f}{P_{in}}, \tag{15}$$

to provide a measure of the amount of recirculating power. The maximum fusion energy occurs when each D ion fuses with each T ion

$$E_{f_{max}} = \frac{n}{2} E_{DT} \pi a^2 L. \tag{16}$$

The actual energy released during the dwell time, $\tau_{flow} = L/v_z$, of the flowing plasma is

$$E_f = \frac{n^2}{4} \tau_{flow} \langle \sigma v \rangle_{DT} E_{DT} \pi a^2 L. \tag{17}$$

The fraction of fuel fused is called the burn fraction and is given by

$$f_b = \frac{E_f}{E_{f_{max}}} = \frac{n}{2} \tau_{flow} \langle \sigma v \rangle_{DT}. \tag{18}$$

The Z-pinch should be long enough to obtain a high burn fraction; however, burn fractions greater than unity are not physically meaningful.

A flow-stabilized Z-pinch reactor design can be shown by scaling the same starting plasma as described previously I_p = 50 kA, T = 20 eV, a = 10 mm, and $n = 6\times10^{22}$ m^{-3}. The resulting parameters are shown in Fig. 7. As a point design, if the plasma current is increased to 1.5 MA, the resulting plasma parameters become T = 18 keV, $n = 1.7\times10^{27}$ m^{-3}, L = 0.75 m, and a = 61 μm. The resulting plasma produces a fusion power of 4.6 TW and Q = 33, again assuming the linear density N remains constant. Note the plasma current can exceed the Pease-Braginskii current because the primary heating mechanism is adiabatic compression and not Ohmic heating. The Ohmic heating power is given by

$$P_\Omega = I^2 R = I^2 \eta \frac{L}{\pi a^2}, \tag{19}$$

where η is the plasma resistivity.

The reactor designs represented in Fig. 7 are not optimized. Such a large fusion power is not desirable. In addition, a more complete analysis is required, but these calculations provide a basis for further investigations.

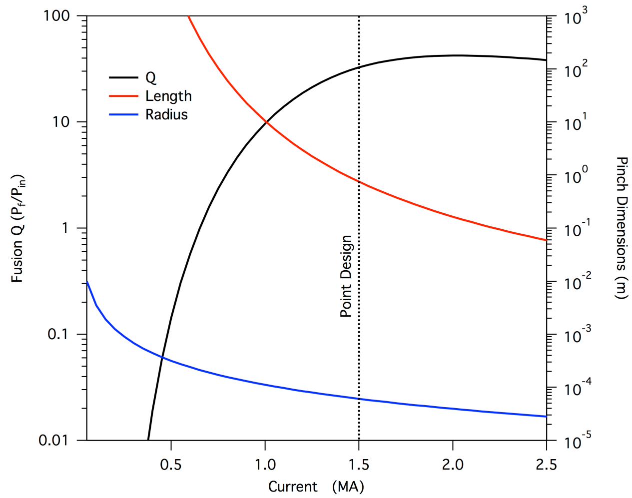

Fig. 7. Plasma and fusion parameters scaled from an initial plasma described by I_p = 50 kA, T = 20 eV, a = 10 mm, and $n = 6\times10^{22}$ m^{-3}. A non-optimized point design is shown as an example.

IV. CONCLUSIONS

The sheared-flow stabilized Z-pinch produces a long-lived plasma that offers intriguing possibilities for scaling to HEDP and even to fusion reactor conditions. While the open ends of the plasma still make contact with the electrodes, the fractional area of the plasma is small. Experimental and computational investigations demonstrate that the surrounding conducting wall is not needed for stability. Therefore, the surrounding wall can be placed arbitrarily far from the plasma. The scaling relations derived in this article show that a experimentally realizable plasma can be scaled to HEDP and fusion reactor parameters. While the calculations are far from optimized, they show that a fusion reactor based on the sheared-flow stabilized Z-pinch is possible and warrants further investigation.

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