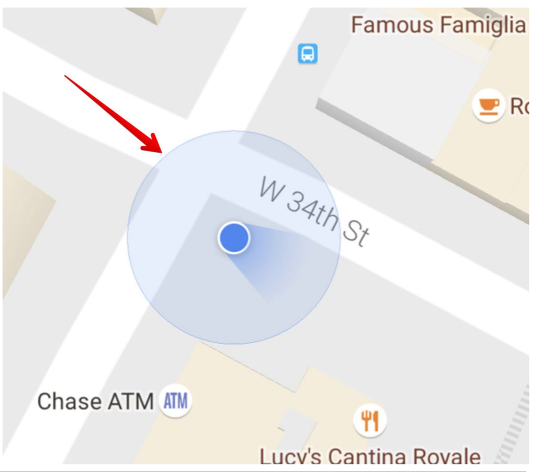
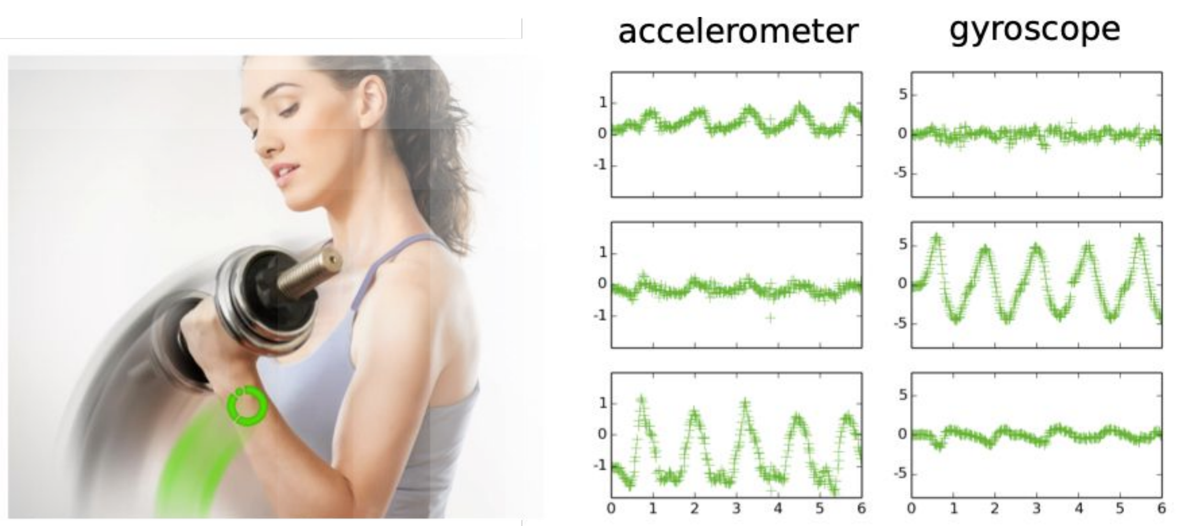
Optimal Transport Particle Filters

STUDENT: Mohammad Al-Jarrah

Embracing Uncertainty in Control Systems



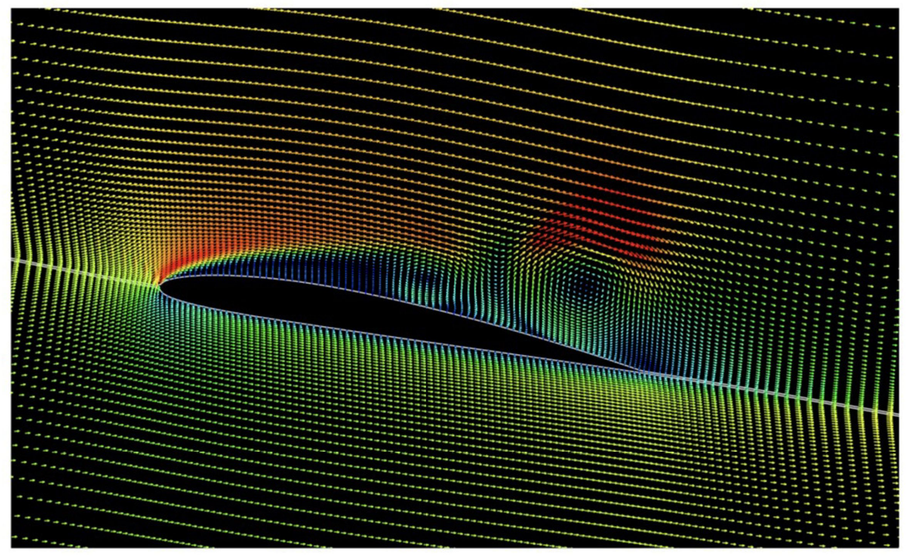
Gesture recognition

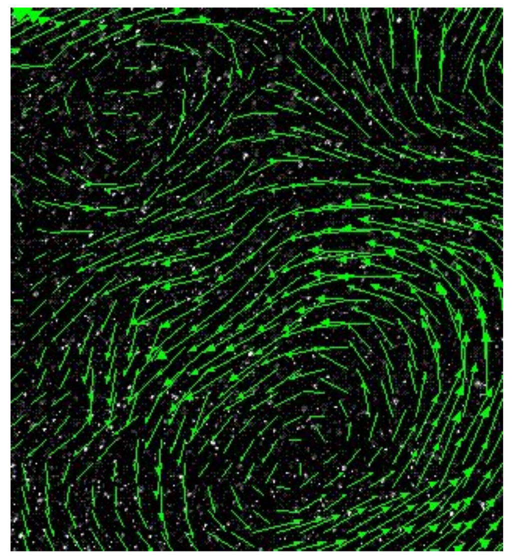


State X_t Observation Z_t

- **Hidden state:** Motion of hand
- **Measurements:** Motion sensors, accelerometer, and gyroscope
- **Problem:** Detection of gestures in real time

Particle image velocimetry





- **Hidden state:** Particle position/ velocity field
- **Measurements:** 2-D slice image of particle tracers
- **Problem:** Estimate velocity and trajectory of moving particle

Nonlinear Filtering, Bayesian inference

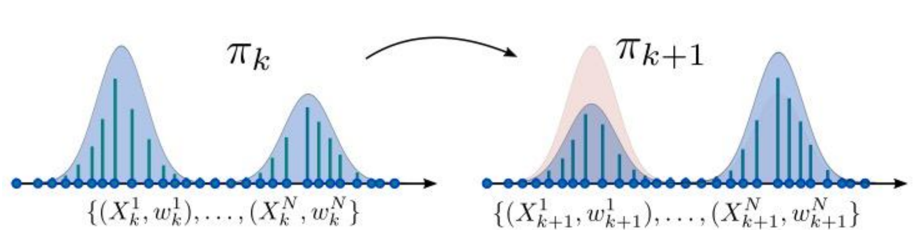
$$\begin{array}{ccccccc} \longrightarrow & X_{k-1} & \longrightarrow & X_k & \longrightarrow & X_{k+1} & \longrightarrow \\ & \downarrow & & \downarrow & & \downarrow & \\ & Y_{k-1} & & Y_k & & Y_{k+1} & \end{array}$$

- State process: $X_k \sim a(\cdot|X_{k-1}), \quad X_0 \sim \pi_0$
- Observation process: $Y_k \sim h(\cdot|X_k)$

Objective: Compute the conditional probability distribution (posterior) $\mathbb{P}(X_t|Y_1,\ldots,Y_t)$

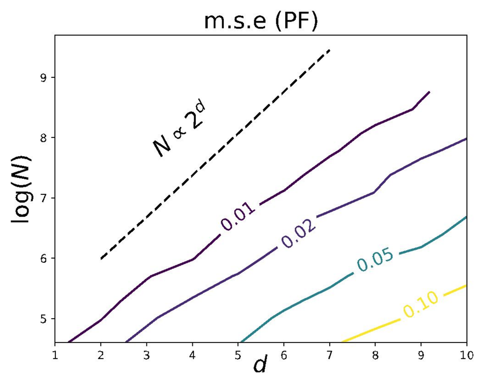
Particle Filter

- Approximate π_k with weighted empirical distribution of particles
- Apply the update rule to the particles and weights



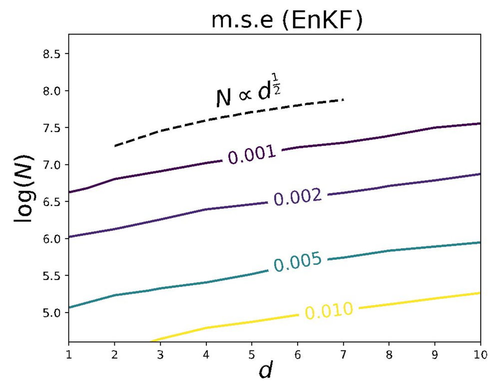
Properties:

- Exact in the limit as N goes to ∞
- Weight degeneracy (curse of dimensionality)



Optimal Transport Particle Filter

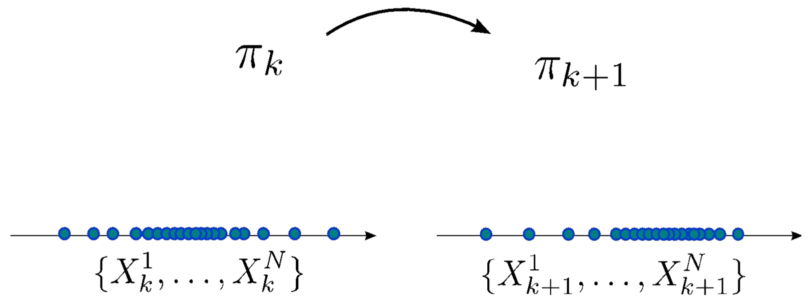
- **Ensemble Kalman filter** avoids curse of dimensionality in linear Gaussian setting
- Can we extend this to **Non-Gaussain** setting?
- Approximate π_k with empirical distribution of particles
- **Main task:**



$$given: \quad \{X_k^1,\ldots,X_k^N\} \sim \pi_k$$

$$generate: \quad \{X_{k+1}^1,\ldots,X_{k+1}^N\} \sim \pi_{k+1}$$

$$where \quad \pi_k = P_{X|Y_{1:k}}$$



- **OTPF approach:** update particle with the optimal transport map form π_k to π_{k+1}

$$X_{k+1}^i = T_k(X_k^i)$$

Optimal Transport formulation of the Bayes Law

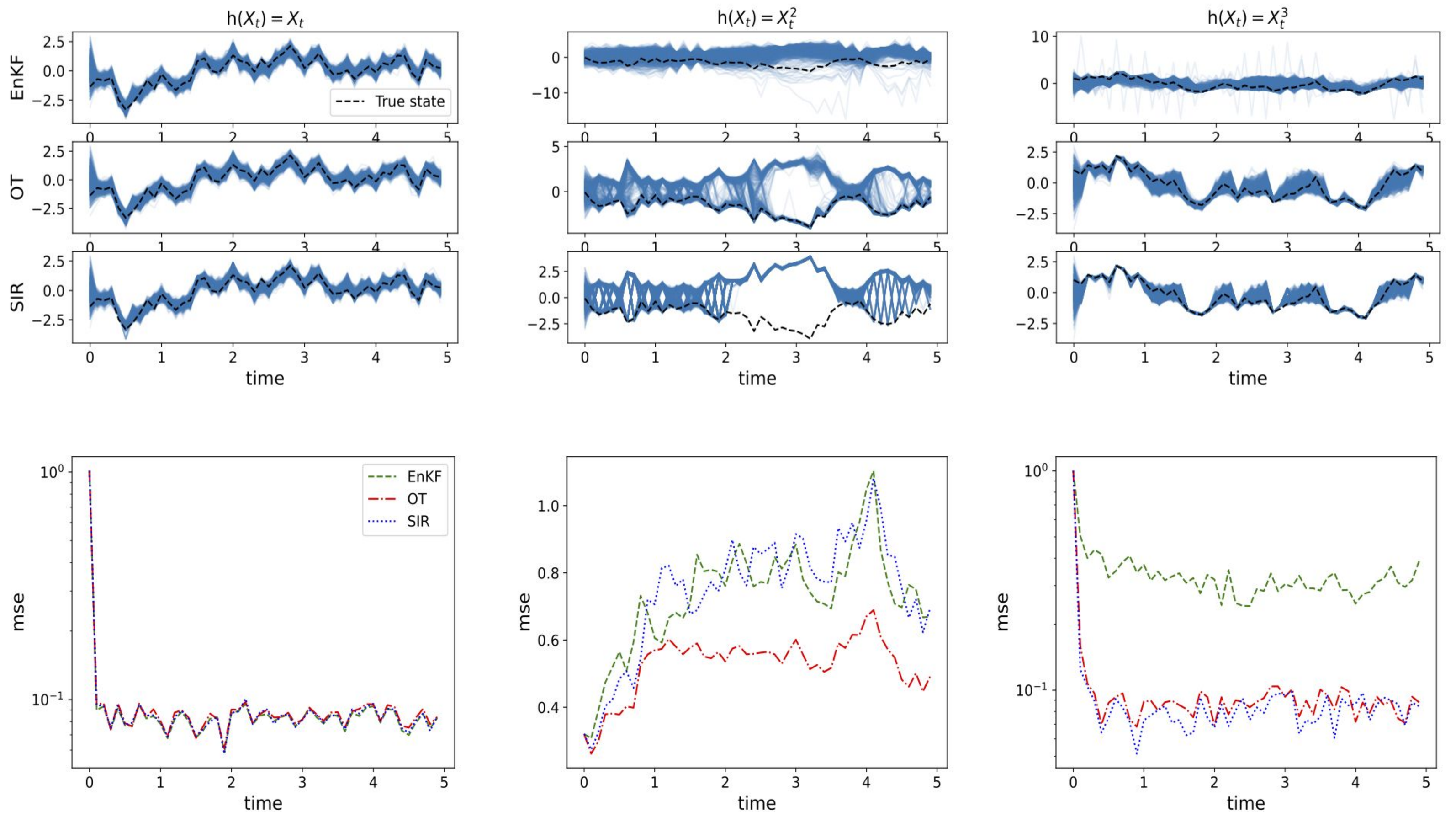
Bayes Law: $P(X|Y) = \dfrac{P(X)P(y|X)}{P(Y)}$

$$= \nabla_x \bar{f}(\cdot;Y)_\# P_x$$

$$where \quad \bar{f} = argmin_{f\in L^1 X\times Y}\mathbb{E}_{(X,Y)\sim P_X\otimes P_Y}[f(X;Y)] + \mathbb{E}_{(X,Y)\sim P_{XY}}[f^*(X;Y)]$$

- Only requires samples $(X_i, Y_i) \sim P_{XY}$ (data-driven / simulation based)
- Enable construction of "approximate" posterior distribution
- Allow application of ML tools (Stochastic optimization and Neural Networks)

Numerical Experiment



Future directions of research

- Establish theoretical guarantees and error bounds
- Efficient representations of the transport map
- Validation on high-dimensional applications

References

- Taghvaei, Amirhossein, and Bamdad Hosseini. "An optimal transport formulation of bayes' law for nonlinear filtering algorithms." 2022 IEEE 61st Conference on Decision and Control (CDC). IEEE, 2022.
- Al-Jarrah, Mohammad, Bamdad Hosseini, and Amirhossein Taghvaei. "Optimal Transport Particle Filters." arXiv preprint arXiv:2304.00392 (2023).

ADVISERS: Amirhossein Taghvaei, Bamdad Hosseini

WILLIAM E. BOEING
DEPARTMENT OF AERONAUTICS & ASTRONAUTICS