Rogue waves in mechanical metamaterials

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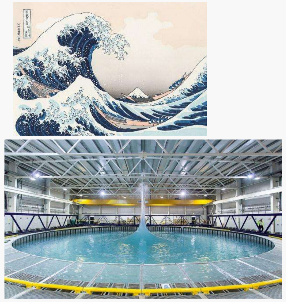
Introduction

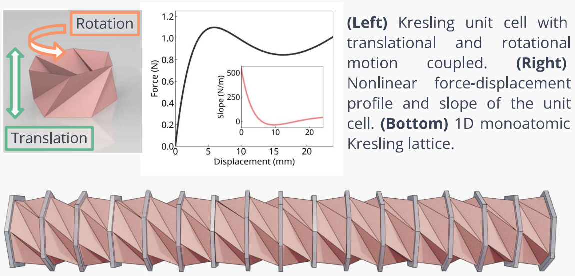
Rogue waves are extremely large waves that appear out of nowhere and disappear without any trace. They have primarily been found in the ocean as hazardous phenomena, which are often referred to as the "wall of the water" or the "holes in the sea" [1]. Despite the vast amount of recent activity on the study of rogue waves, there have been relatively **few reports** on their study in **solids or structures**, and in the associated spatially discrete models [2]. Furthermore, especially, their **experimental observation** remains very elusive.



(Top) Rogue waves in the ocean (Hokusai, The Great Wave off Kanagawa) and **(bottom)** water tank (© The University of Edinburgh).

Objectives

- Explore the **theoretical framework** for the rogue wave prediction in the origami-inspired mechanical system.
- Numerically **verify theoretical study** and **predict** the mechanical rogue waves of a general multi-DOF lattice.



(Left) Kresling unit cell with translational and rotational motion coupled. **(Right)** Nonlinear force-displacement profile and slope of the unit cell. **(Bottom)** 1D monoatomic Kresling lattice.

Theoretical framework

Equations of motion of Kresling lattice

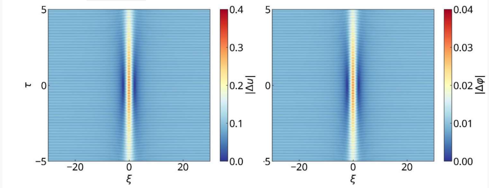
$$m_n \ddot{u}_n + F_n(u_n - u_{n+1}, \varphi_n - \varphi_{n+1}) - F_{n-1}(u_{n-1} - u_n, \varphi_{n-1} - \varphi_n) = 0, \quad \text{(Translation)}$$

$$j_n \ddot{\varphi}_n + T_n(u_n - u_{n+1}, \varphi_n - \varphi_{n+1}) - T_{n-1}(u_{n-1} - u_n, \varphi_{n-1} - \varphi_n) = 0, \quad \text{(Rotation)}$$

Axial force (F_{n-1}); Torque (T_{n-1})

⇩ Multiple scale analysis

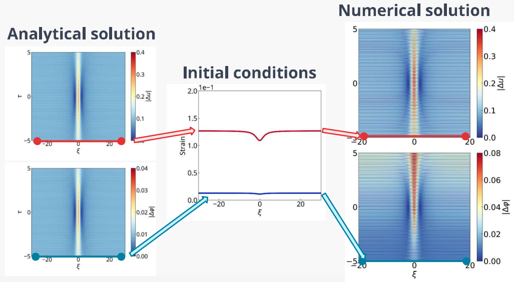
2DOF nonlinear Schrödinger equations (2DOF-NLSE) ⇨

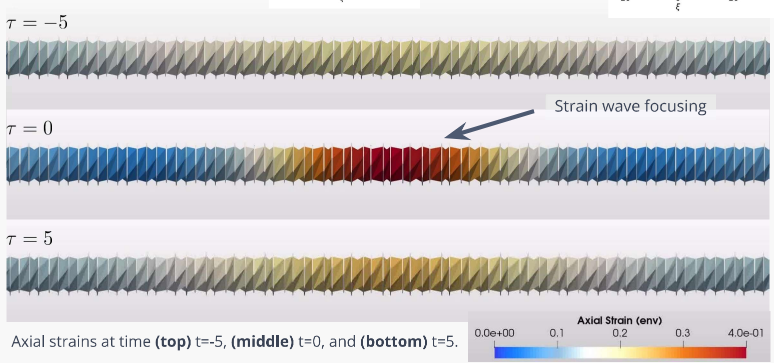


Analytically predicted rogue waves as exact solutions to 2DOF NLSE. **(Left)** translational component, **(right)** rotational component.

Rogue wave results (numerical)

We use analytically predicted rogue wave solutions to conduct **numerical simulation**. The numerical solution **agrees** very well with the analytical solution except for a small discrepancy in the rotational component.





Axial strains at time **(top)** t=-5, **(middle)** t=0, and **(bottom)** t=5.

Conclusions and Future Works

Conclusions

In conclusion, we have analytically and numerically explored nonlinear waves in a lattice with translational and rotational components coupled. Specifically, we have:

- Introduced **theoretical framework** for obtaining rogue wave solutions for coupled lattice, e.g., Kresling origami lattice.
  - Multiple scale analysis yields the NLS-type coupled equations with exact solutions.

2DOF-NLSE

$$i\partial_\tau A_{1,1} + \nu_2 \partial_\xi^2 A_{1,1} + \nu_3 \partial_\xi^2 B_{1,1} + \nu_4 |A_{1,1}|^2 A_{1,1} + \nu_5 |B_{1,1}|^2 B_{1,1} + \nu_6 |B_{1,1}|^2 A_{1,1} + \nu_7 |A_{1,1}|^2 B_{1,1} + \nu_8 B_{1,1}^* A_{1,1}^2 + \nu_9 A_{1,1}^* B_{1,1}^2 = 0$$

$$i\partial_\tau B_{1,1} + \mu_2 \partial_\xi^2 A_{1,1} + \mu_3 \partial_\xi^2 B_{1,1} + \mu_4 |A_{1,1}|^2 A_{1,1} + \mu_5 |B_{1,1}|^2 B_{1,1} + \mu_6 |B_{1,1}|^2 A_{1,1} + \mu_7 |A_{1,1}|^2 B_{1,1} + \mu_8 B_{1,1}^* A_{1,1}^2 + \mu_9 A_{1,1}^* B_{1,1}^2 = 0$$

- Verified the analytical prediction by comparing it with the **numerical simulation** of the lattice dynamics.

Future works

- **Experimental verification** of the mechanical rogue waves using the finite lattice.
  - Not limited to Kresling origami, but also other nonlinear mechanical metamaterials.

References

1. Christian Kharif and Efim Pelinovsky. Physical mechanisms of the rogue wave phenomenon. European Journal of Mechanics - B/Fluids, 22(6):603–634, 2003.
2. E. G. Charalampidis, J. Lee, P. G. Kevrekidis, and C. Chong. Phononic rogue waves. Physical Review E, 98(3):032903, 2018.

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