Rogue waves in mechanical metamaterials
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Introduction

Rogue waves are extremely large waves that appear out of nowhere and disappear without any trace. They have primarily been found in the ocean as hazardous phenomena, which are often referred to as the “wall of the water” or the “holes in the sea” [1]. Despite the vast amount of recent activity on the study of rogue waves, there have been relatively few reports on their study in solids or structures, and in the associated spatially discrete models [2]. Furthermore, especially, their experimental observation remains very elusive.

Objectives

- Explore the theoretical framework for the rogue wave prediction in the origami-inspired mechanical system.

- Numerically verify theoretical study and predict the mechanical rogue waves of a general multi-DOF lattice.

Theoretical framework

Equations of motion of Kresling lattice

\[ m_n \ddot{u}_n + F_n (u_n - u_{n+1}, \dot{u}_n - \dot{u}_{n+1}) = F_{n-1} (u_{n-1} - u_n, \dot{u}_{n-1} - \dot{u}_n) = 0, \]

\[ j_n \ddot{\phi}_n + T_n (u_n - u_{n+1}, \dot{\phi}_n - \dot{\phi}_{n+1}) = T_{n-1} (u_{n-1} - u_n, \dot{\phi}_{n-1} - \dot{\phi}_n) = 0, \]

Multiple scale analysis

2DOF nonlinear Schrödinger equations (2DOF-NLSE)

Analytically predicted rogue waves as exact solutions to 2DOF-NLSE: (Left) translational component, (Right) rotational component.

Rogue wave results (numerical)

We use analytically predicted rogue wave solutions to conduct a numerical simulation. The numerical solution agrees very well with the analytical solution except for a small discrepancy in the rotational component.

Conclusions and Future Works

Conclusions

In conclusion, we have analytically and numerically explored nonlinear waves in a lattice with translational and rotational components coupled. Specifically, we have:

- Introduced theoretical framework for obtaining rogue wave solutions for coupled lattice, e.g., Kresling origami lattice.

  - Multiple scale analysis yields the NLS-type coupled equations with exact solutions:

    \[ i \hbar \frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial t^2} + \rho A (|A|^2 A) + m \rho A^2 A^* = 0, \]

    \[ i \hbar \frac{\partial B}{\partial z} + \frac{\partial^2 B}{\partial t^2} + \rho B (|B|^2 B) + m \rho B^2 B^* = 0, \]

    \[ \text{2DOF-NLSE} \]

  - Verified the analytical prediction by comparing it with the numerical simulation of the lattice dynamics.

Future works

- Experimental verification of the mechanical rogue waves using the finite lattice.

  - Not limited to Kresling origami, but also other nonlinear mechanical metamaterials.

References


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