Deferred Decision Trajectory Optimization

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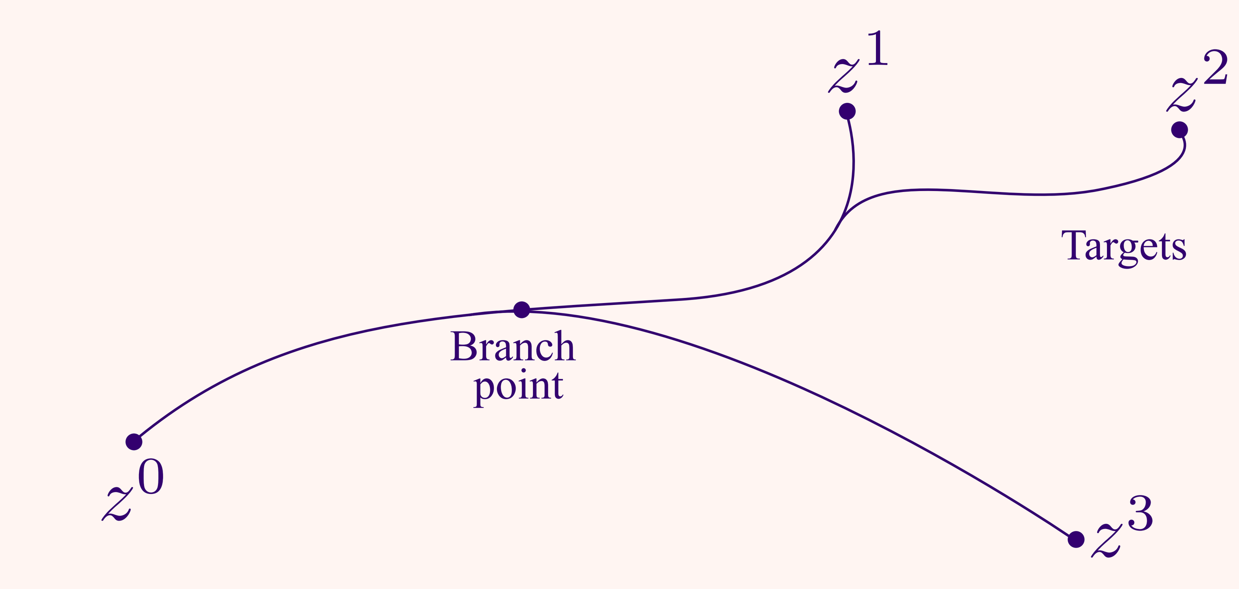
Deferring decision for robust mission planning

- Deferred decision trajectory optimization (DDTO) is a **deterministic** framework for ensuring robustness to uncertainties by generating trajectories that keep multiple candidate targets accessible for as long as possible while also **respecting constraints due to fuel consumption** or control effort.
- In many practical scenarios, the vehicle might **lack complete information about its environment**. So, it is desirable to **defer decision to choose a target** until more reliable information about the environment is acquired.

Multi-target trajectory optimization

A **branch point** is the farthest state (in time) until which all candidate targets are accessible.



Delay branch point by quasiconvex optimization

Given n targets, the **problem of delaying a branch point** for as long as possible is a **quasiconvex optimization** problem which can be **solved via bisection search**.

$$\begin{array}{lll} \underset{\substack{t\in\mathbb{N},\\ u^j_\tau,\ \tau=0,\ldots,N-1,\ j=1,2}}{\text{maximize}} & t & \\ \text{subject to} & x^j_\tau = A_d x^j_{\tau-1} + B_d u^j_{\tau-1}, & j=1,2,\ \tau=1,\ldots,N,\\ & u^j_\tau \in \mathbb{U}, & j=1,2,\ \tau=0,\ldots,N-1,\\ & u^1_\tau = u^2_\tau, & \tau=0,\ldots,t-1,\\ & x^j_0 = z^0, & j=1,2,\\ & x^j_N = z^j, & j=1,2. \end{array}$$

Maximize **quasiconcave** function:

$$g(U^1,U^2) = \max\{\, t \mid x^1_\tau = x^2_\tau,\ \text{for } \tau=1,\ldots,t\}$$

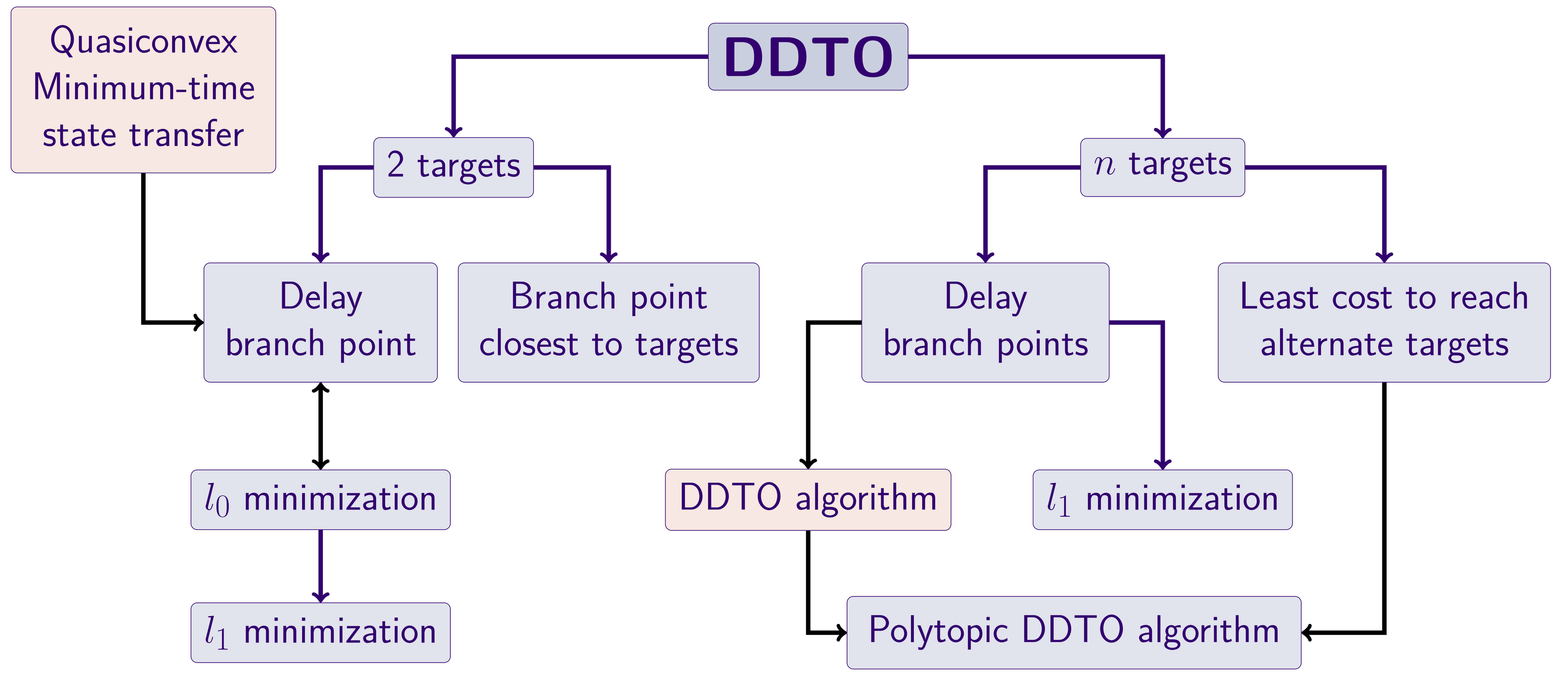
$U^j = (u^j_0,\ldots,u^j_{N-1})$, for $j=1,2$.

The problem above is inspired by the minimum-time state transfer problem

$$\begin{array}{lll} \underset{u_\tau,\ \tau=0,\ldots,N-1}{\text{minimize}} & t & \\ \text{subject to} & x_\tau = A_d x_{\tau-1} + B_d u_{\tau-1}, & \tau=1,\ldots,N,\\ & u_\tau \in \mathbb{U}, & \tau=0,\ldots,N-1,\\ & x_0 = z^0, & \\ & x_\tau = z^1, & \tau=t,\ldots,N. \end{array}$$

Minimize **quasiconvex** function:

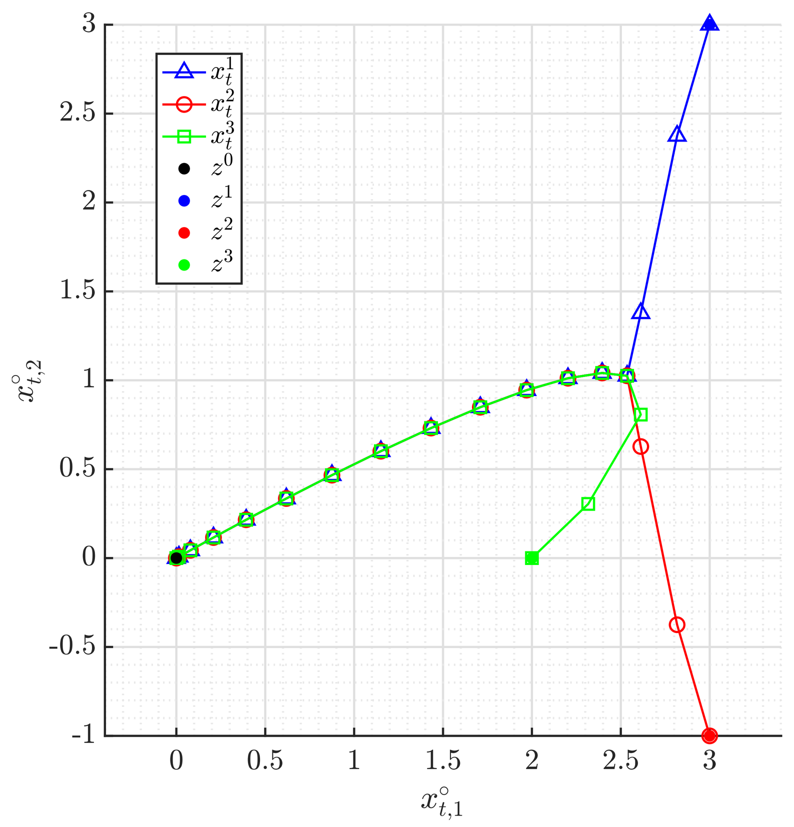
$$f(u_0,\ldots,u_{N-1}) = \min\{\, t \mid x_\tau = z^1,\ \text{for } \tau=t,\ldots,N\}$$

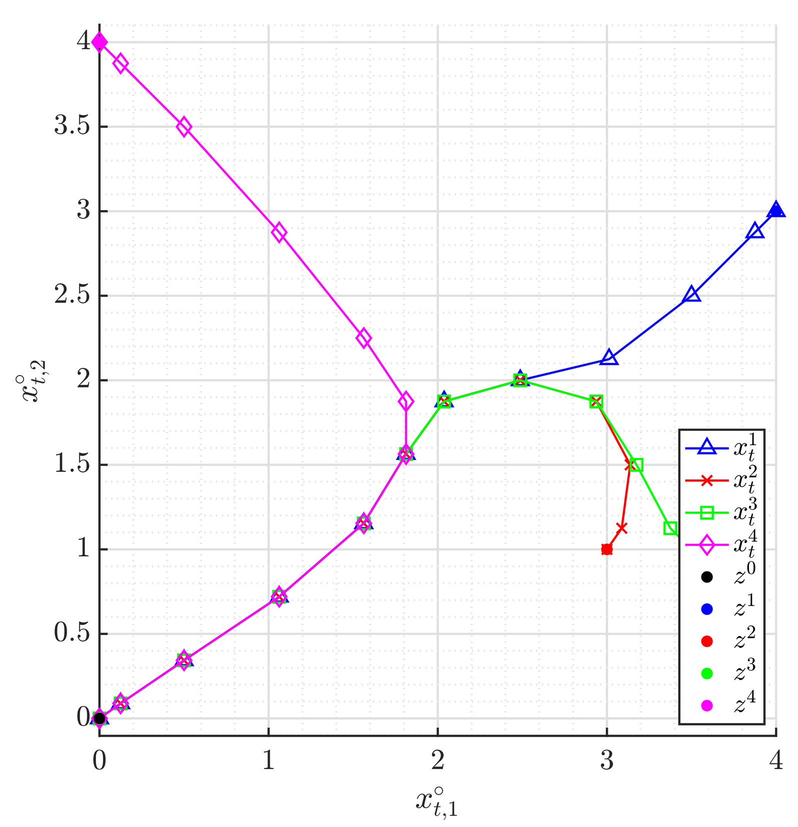


Sparse minimization

- The **problem of delaying the branch point** associated with trajectories to two targets is **equivalent to** the problem of **minimizing the l_0 norm of the difference between the states** of the two trajectories at each time instant.
- For the n-target case this connection is slightly weaker. Generating a **hierarchy of branch points** for subset of targets is similar to **minimizing the l_1 norm of the difference between states** of any two trajectories.

$$\begin{array}{lll} \text{minimize} & \displaystyle\sum_{\substack{i,j=1\\ i\neq j}}^{n} \|x^j_t - x^i_t\|_1 & \\ \text{subject to} & x^j_{t+1} = A_d x^j_t + B_d u^j_t, & t=0,\ldots,N-1,\ j=1,\ldots,n\\ & x^j_{t+1}\in\mathbb{X},\ u^j_t\in\mathbb{U}, & t=0,\ldots,N-1,\ j=1,\ldots,n\\ & x^j_0 = z^0,\ x^j_N = z^j, & j=1,\ldots,n. \end{array}$$





DDTO Algorithm

We can generate a hierarchy of branch points by **recursively delaying branch points** for a shrinking set of targets.

Data: Target set J, target preference order $\{\lambda_k\}_{k=1}^{n-2}$, initial state z^0, target states z^j, suboptimality fractions ϵ^j, and horizon lengths N^j for $j\in J$

Result: Branch points $\{z_k\}_{k=1}^{n-1}$

Initialize $k=0$, first branch point $z_0 = z^0$, first target set $J_0 = J$

while J_k *contains more than* 2 *targets* **do**

estimate z_{k+1}; /* maximize g for J_k with ϵ^j suboptimality for $j\in J_k$ */

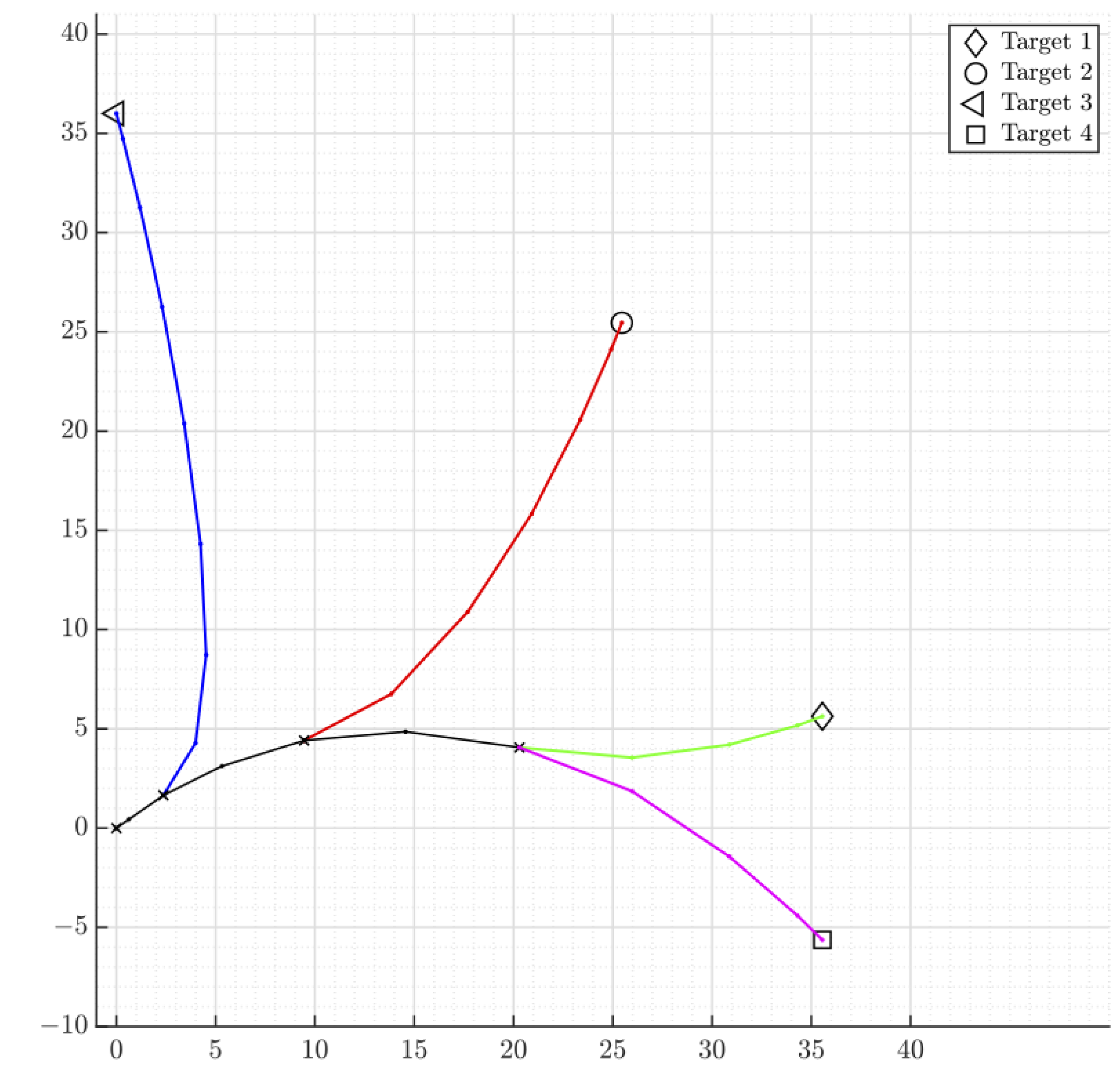
$J_{k+1} \leftarrow J_k$; /* reject target */

$k \leftarrow k+1$;

end

Nonlinear dynamics with nonconvex constraints

DDTO for 2D double integrator with (nonlinear) drag force and obstacle avoidance constraints.



References

[1] Purnanand Elango, Selahattin Burak Sarsılmaz, and Behçet Açıkmeşe. Deferring Decision in Multi-target Trajectory Optimization. In *AIAA SciTech Forum 2022*, Jan 2022.

[2] Purnanand Elango, Selahattin Burak Sarsılmaz, and Behçet Açıkmeşe. Deferred-Decision Trajectory Optimization.