Trace – a game-changing discovery

Stephen W. Tsai
Stanford University

October 5, 2015
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
How Many Data Points?

• In netting analysis that is applied to pressure vessels and pipes, fiber strength is the one and only design validation calculated from burst pressure: the answer is one.

• Carbon fibers are known to be anisotropic but we know only the longitudinal fiber stiffness and strength: the answer is one as before, plus one for stiffness.

• With trace, laminates become universal.
Dominant “11” Component in 2- & 3-D

Trace-normalized stiffness: Universal [0]

2D: Plane stress

Q_{11}^* = 0.885; 1.3%

E_1^* = 0.880; 1.3%

\frac{0.880}{0.735} = 1.20 = 3D/2D

3D: Plane strain

C_{11}^* = 0.752; 2.1%

E_1^* = 0.735; 2.1%
Material Selection for Weight Savings

The graph illustrates the trade-off between weight and stiffness for various materials. Different categories are indicated:

- **Strength-controlled**
- **Stiffness-controlled**

A built-in safety factor due to stiffness scaling is also noted. The trace, GPa, and safety factor are plotted against various materials, including:

- T700 C-Ply 55
- AS4/MTM45
- T700/2510
- T4708/MR60H
- T650/epoxy
- T300/IM7/977-3
- IM7/8552
- IM7/IM6/epoxy

The graph helps in selecting materials that balance weight savings with sufficient structural integrity for different applications.
Relative Cost of Hybrid Laminates

Linear correlation with trace, but not with $E_x$ or others

$$\text{Trace}^{\text{Hybrid}} = v_1 \text{Trace}_1 + v_2 \text{Trace}_2 + \ldots$$
Black Aluminum vs Trace: 15 CFRP

\[ E_x = 0.88 \text{ Trace} \]

Black aluminum vs Trace

Ply stiffness

\[ E_y, V_x, E_s \]

Unit circle

Universal Laminates

So much simpler: 4:1 5:2 15:1 15:1 ---- 15:1
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
Tensors and Traces for Composites

- Stress components $\sigma_i$
- Strain components $\varepsilon_i$
- Stiffness: $Q_{ij}, A_{ij}, D_{ij}, C_{ij},...$
- Failure criterion in stress space: $F_{ij}, F_i$
  where $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$
- Failure criterion in strain space: $G_{ij}, G_i$
  where $G_{ij}\varepsilon_i\varepsilon_j + G_i\varepsilon_i = 1$
  $G_{ij} = Q_{ik}Q_{jl}F_{kl}, G_i = Q_{ij}F_j$

\[
\begin{align*}
\text{Tr } [\sigma] &= \sigma_1 + \sigma_2 + \sigma_3 = \text{pressure} \\
\text{Tr } [\varepsilon] &= \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \Delta \text{ volume} \\
\text{Tr } [Q] &= Q_{11} + Q_{22} + 2Q_{66} \\
&= \text{Tr } [A^*] = A_{11}^* + A_{22}^* + 2A_{66}^* \\
&= \text{Tr } [D^*] = D_{11}^* + D_{22}^* + 2D_{66}^* \\
\text{Tr } [C] &= C_{11} + C_{22} + C_{33} +...+ 2C_{66} \\
\text{Tr } [F] &= F_{11} + F_{22} + F_{66}/2 \\
\text{Tr } \{F\} &= F_1 + F_2 \\
\text{Tr } [G] &= G_{11} + G_{22} + 2G_{66} \\
\text{Tr } \{G\} &= G_1 + G_2
\end{align*}
\]
## Evolution: Universal Constants [0] CFRP

### Ply engineering constants

<table>
<thead>
<tr>
<th>Ply material</th>
<th>$E_1$</th>
<th>$E_2$</th>
<th>$E_6$</th>
<th>$\nu_{21}$</th>
<th>$Q_{11}$</th>
<th>$Q_{22}$</th>
<th>$Q_{21}$</th>
<th>$Q_{66}$</th>
<th>Trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM6/epoxy</td>
<td>203</td>
<td>11.2</td>
<td>8.4</td>
<td>0.32</td>
<td>204</td>
<td>11.3</td>
<td>3.6</td>
<td>8.4</td>
<td>232</td>
</tr>
<tr>
<td>IM7/977-3</td>
<td>191</td>
<td>9.9</td>
<td>7.8</td>
<td>0.35</td>
<td>192</td>
<td>10.0</td>
<td>3.5</td>
<td>7.8</td>
<td>218</td>
</tr>
<tr>
<td>T300/5208</td>
<td>181</td>
<td>10.3</td>
<td>7.2</td>
<td>0.28</td>
<td>182</td>
<td>10.3</td>
<td>2.9</td>
<td>7.2</td>
<td>206</td>
</tr>
<tr>
<td>IM7/MTM45</td>
<td>175</td>
<td>8.2</td>
<td>5.5</td>
<td>0.33</td>
<td>176</td>
<td>8.2</td>
<td>2.7</td>
<td>5.5</td>
<td>195</td>
</tr>
<tr>
<td>T800/Cytec</td>
<td>162</td>
<td>9.0</td>
<td>5.0</td>
<td>0.40</td>
<td>163</td>
<td>9.1</td>
<td>3.6</td>
<td>5.0</td>
<td>183</td>
</tr>
<tr>
<td>IM7/8552</td>
<td>159</td>
<td>9.0</td>
<td>5.5</td>
<td>0.32</td>
<td>160</td>
<td>9.0</td>
<td>2.9</td>
<td>5.5</td>
<td>180</td>
</tr>
<tr>
<td>T800S/3900</td>
<td>151</td>
<td>8.2</td>
<td>4.0</td>
<td>0.33</td>
<td>152</td>
<td>8.2</td>
<td>2.7</td>
<td>4.0</td>
<td>168</td>
</tr>
<tr>
<td>T300/F934</td>
<td>148</td>
<td>9.7</td>
<td>4.6</td>
<td>0.30</td>
<td>149</td>
<td>9.7</td>
<td>2.9</td>
<td>4.6</td>
<td>168</td>
</tr>
<tr>
<td>T700 C-Ply 64</td>
<td>141</td>
<td>9.3</td>
<td>5.8</td>
<td>0.30</td>
<td>142</td>
<td>9.4</td>
<td>2.8</td>
<td>5.8</td>
<td>163</td>
</tr>
<tr>
<td>AS4/H3501</td>
<td>138</td>
<td>9.0</td>
<td>7.1</td>
<td>0.30</td>
<td>139</td>
<td>9.0</td>
<td>2.7</td>
<td>7.1</td>
<td>162</td>
</tr>
<tr>
<td>T650/epoxy</td>
<td>139</td>
<td>9.4</td>
<td>5.5</td>
<td>0.32</td>
<td>140</td>
<td>9.5</td>
<td>3.0</td>
<td>5.5</td>
<td>160</td>
</tr>
<tr>
<td>T4708/MR60H</td>
<td>142</td>
<td>7.7</td>
<td>3.8</td>
<td>0.34</td>
<td>143</td>
<td>7.8</td>
<td>2.6</td>
<td>3.8</td>
<td>158</td>
</tr>
<tr>
<td>T700/2510</td>
<td>126</td>
<td>8.4</td>
<td>4.2</td>
<td>0.31</td>
<td>127</td>
<td>8.5</td>
<td>2.6</td>
<td>4.2</td>
<td>144</td>
</tr>
<tr>
<td>AS4/MTM45</td>
<td>127</td>
<td>7.9</td>
<td>3.6</td>
<td>0.30</td>
<td>128</td>
<td>8.0</td>
<td>2.4</td>
<td>3.6</td>
<td>143</td>
</tr>
<tr>
<td>T700 C-Ply 55</td>
<td>121</td>
<td>8.0</td>
<td>4.7</td>
<td>0.30</td>
<td>122</td>
<td>8.0</td>
<td>2.4</td>
<td>4.7</td>
<td>139</td>
</tr>
</tbody>
</table>

### Plane-stress stiff matrix

| Trace-normalized |
|------------------|------------------|------------------|------------------|
| $E_1^*$          | 0.874            | 0.877            | 0.877            |
| $E_2^*$          | 0.048            | 0.046            | 0.050            |
| $E_6^*$          | 0.036            | 0.036            | 0.035            |

### Trace

$\text{Trace} = Q_{11} + Q_{22} + 2Q_{66}$

### Average

<table>
<thead>
<tr>
<th>Average = Universal</th>
<th>0.320</th>
<th>1.000</th>
<th>0.880</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std dev</td>
<td>0.029</td>
<td>0.013</td>
<td>0.052</td>
</tr>
</tbody>
</table>
E\textsubscript{1\_master ply, GPa}:
- CFRP tape tens: 0.423; 1.3%
- CFRP fabric tens: 0.471; 1.2%
- GFRP fabric tens:
  - RT-dry, cold-dry, hot-dry, hot-wet: 0.471; 1.2%
  - Good correlation
- CFRP tape compr: 0.888; 2.1%
- CFRP fabric compr: 0.472; 1.3%
- GFRP fabric compr:
  - RT-dry, cold-dry, hot-dry, hot-wet: 0.472; 1.3%
  - Good correlation
- Multiple CFRP, GFRP:
  - RT-dry, cold-dry, hot-dry, hot-wet - 2%
- CFRP braid tens: 0.423; 1.3%
- CFRP braid compr: 0.433; 1.5%
- GFRP fabric compr:
  - RT-dry, cold-dry, hot-dry, hot-wet: 0.433; 1.5%
  - Good correlation
  - No correlation
Composition of Trace: Carbon, Kev, Glass

- Carbon/epoxy: (88, 5, 3)%
- Kevlar 49/epoxy: (88, 6, 3)%
- E-glass/epoxy: (70, 15, 7)%

<table>
<thead>
<tr>
<th>Ply material</th>
<th>$Q_{xx}^*$</th>
<th>$Q_{yy}^*$</th>
<th>$Q_{ss}^*$</th>
<th>$Q_{xy}^*$</th>
<th>Trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average CFRP</td>
<td>0.88</td>
<td>0.05</td>
<td>0.03</td>
<td>0.02</td>
<td>1.000</td>
</tr>
<tr>
<td>Kevlar/epoxy</td>
<td>0.88</td>
<td>0.06</td>
<td>0.03</td>
<td>0.03</td>
<td>1.000</td>
</tr>
<tr>
<td>E-glass/epoxy</td>
<td>0.70</td>
<td>0.15</td>
<td>0.07</td>
<td>0.04</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Universal ply: Transversely isotropic [0]

- Fiber controlled:
  - In-plane: $Q_{11} = 88\%$
  - Out-of-plane: $Q_{22} + 2Q_{66} = 12\%$

- Matrix controlled:
  - In-plane: $C_{11} = 75\%$
  - Out-of-plane: $C_3 + 2(C_{44} + C_{55}) = 14\%$

- Sub-trace (in-plane) = $C_{11} + C_{22} + 2C_{66} = 86\%$
- Sub-trace (out-of-plane) = $C_{33} + 2(C_{44} + C_{55}) = 14\%$

Trace 2D:
- $Q_{11} = 88\%$
- $Q_{22} + 2Q_{66} = 12\%$

Trace 3D:
- $Q_{11} = 88\%$
- $Q_{22} + 2Q_{66} = 12\%$

2D Stress:
- Fiber
- Matrix

3D Stress:
- Fiber
- Matrix

- In-plane
  - $Q_{11} = 88\%$
- Out-of-plane
  - $Q_{22} + 2Q_{66} = 12\%$

Transverse shear:
- $C_3 + 2(C_{44} + C_{55}) = 14\%$

Transverse stiffness:
- $C_{11} + C_{22} + 2C_{66} = 86\%$
$E_x$ of $[0]$ 

$E_{1\circ}$ of $[\pi/4]$

0.880, cv = 1.33%

0.337, cv = 0.10%
Ascending 2D and 3D Trace: CFRP

<table>
<thead>
<tr>
<th>15 Ply material</th>
<th>2D Tr</th>
<th>3D Tr</th>
<th>3D/2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>T700 C-Ply 55</td>
<td>139</td>
<td>169</td>
<td>1.21</td>
</tr>
<tr>
<td>AS4/MTM45</td>
<td>143</td>
<td>170</td>
<td>1.19</td>
</tr>
<tr>
<td>T700/2510</td>
<td>144</td>
<td>174</td>
<td>1.21</td>
</tr>
<tr>
<td>T4708/MR60H</td>
<td>158</td>
<td>186</td>
<td>1.18</td>
</tr>
<tr>
<td>T650/epoxy</td>
<td>160</td>
<td>196</td>
<td>1.22</td>
</tr>
<tr>
<td>AS4/H3501</td>
<td>162</td>
<td>199</td>
<td>1.23</td>
</tr>
<tr>
<td>T700 C-Ply 64</td>
<td>163</td>
<td>198</td>
<td>1.22</td>
</tr>
<tr>
<td>T300/F934</td>
<td>168</td>
<td>201</td>
<td>1.20</td>
</tr>
<tr>
<td>T800S/3900</td>
<td>168</td>
<td>197</td>
<td>1.17</td>
</tr>
<tr>
<td>IM7/8552</td>
<td>180</td>
<td>214</td>
<td>1.19</td>
</tr>
<tr>
<td>T800/Cytec</td>
<td>183</td>
<td>217</td>
<td>1.19</td>
</tr>
<tr>
<td>IM7/MTM45</td>
<td>195</td>
<td>227</td>
<td>1.17</td>
</tr>
<tr>
<td>T300/5208</td>
<td>206</td>
<td>247</td>
<td>1.19</td>
</tr>
<tr>
<td>IM7/977-3</td>
<td>218</td>
<td>260</td>
<td>1.19</td>
</tr>
<tr>
<td>IM6/epoxy</td>
<td>232</td>
<td>278</td>
<td>1.20</td>
</tr>
</tbody>
</table>

Average: 1.20
Std deviation: 1.7%
Dominant “11” Component in 2- & 3-D

Trace-normalized stiffness: Universal [0]

Q_{11}^* = 0.885; 1.3%

E_1^* = 0.880; 1.3%

C_{11}^* = 0.752; 2.1%

E_1^* = 0.735; 2.1%

2D: Plane stress

3D: Plane strain

\[
\frac{0.880}{0.735} = 1.20
\]

= 3D/2D

\[
\frac{0.880}{0.735} = 1.20
\]
Dominant “11” Component in 2- & 3-D

Trace-normalized stiffness: Universal \([\pi/3],[\pi/4], \ldots\)

**2D: Plane stress**
- \(Q_{11}^* = 0.371; 0.1\%\)
- \(Q_{66}^* = 0.129; 0.1\%\)
- \(E_1^* = E_2^* = 0.336; 0.1\%\)
- \(E_6^* = 0.129; 0.1\%\)

**3D: Plane strain**
- \(A_{11}^* = A_{22}^* = 0.323; 0.5\%\)
- \(A_{66}^* = 0.107; 0.1\%\)
- \(A_{33}^* = 0.057; 0.5\%\)
- \(A_{44}^* = A_{55}^* = 0.020; 0.2\%\)
- \(E_1^* = E_2^* = 0.281; 0.4\%\)
- \(E_6^* = 0.107; 0.1\%\)
- \(E_3^* = 0.053; 0.5\%\)
Components: 2D Universal Laminates

Universal laminate stiffness

E₁* = E₂* = 0.468

[0/90]

E₆* = 0.031

Quasi-iso

E₁* = 0.370

[0/±45]

E₆* = 0.161

E₂* = 0.155

[0/±45/0]

E₆* = 0.129

[0/±45]

E₁* = 0.499

[0/±45/0]

E₂* = 0.141

E₆* = 0.129

2D Trace, GPa

15 Individual CFRP
# Universal Laminate Constants

Generated from classical laminated plate theory, and universal [0] derived from average of 15 CFRP

<table>
<thead>
<tr>
<th>Universal laminates</th>
<th>$E_1^*$</th>
<th>$E_2^*$</th>
<th>$E_6^*$</th>
<th>$\nu_{21}$</th>
<th>Trace</th>
</tr>
</thead>
<tbody>
<tr>
<td>Universal [0]</td>
<td>0.880</td>
<td>0.052</td>
<td>0.031</td>
<td>0.320</td>
<td>1.000</td>
</tr>
<tr>
<td>[0/90]</td>
<td>0.468</td>
<td>0.468</td>
<td>0.031</td>
<td>0.036</td>
<td>1.000</td>
</tr>
<tr>
<td>[$\pi/4$]</td>
<td>0.336</td>
<td>0.336</td>
<td>0.129</td>
<td>0.308</td>
<td>1.000</td>
</tr>
<tr>
<td>[0$_7$/±45/90]</td>
<td>0.662</td>
<td>0.175</td>
<td>0.070</td>
<td>0.310</td>
<td>1.000</td>
</tr>
<tr>
<td>[0/±45$_4$/90]</td>
<td>0.217</td>
<td>0.217</td>
<td>0.187</td>
<td>0.552</td>
<td>1.000</td>
</tr>
<tr>
<td>[0/±45]</td>
<td>0.370</td>
<td>0.155</td>
<td>0.161</td>
<td>0.734</td>
<td>1.000</td>
</tr>
<tr>
<td>[0/±45/0]</td>
<td>0.499</td>
<td>0.141</td>
<td>0.129</td>
<td>0.701</td>
<td>1.000</td>
</tr>
<tr>
<td>[0/±30]</td>
<td>0.510</td>
<td>0.074</td>
<td>0.129</td>
<td>1.220</td>
<td>1.000</td>
</tr>
<tr>
<td>[0/±30/0]</td>
<td>0.611</td>
<td>0.072</td>
<td>0.104</td>
<td>1.079</td>
<td>1.000</td>
</tr>
<tr>
<td>[$\pm12.5$]</td>
<td>0.764</td>
<td>0.053</td>
<td>0.066</td>
<td>0.913</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Components: 3D Universal Laminates

C_{11} = C_{22} = 0.405
C_{11}^* = 0.41

\left[ \frac{\pi}{3} \right], \left[ \frac{\pi}{4} \right], \ldots

In-plane: 0.86
Out-plane: 0.14

C_{66}^* = 0.026
C_{22}^* = 0.41
C_{11}^* = 0.41

[C_{0/90}]

\left[ \frac{\pi}{3} \right], \left[ \frac{\pi}{4} \right], \ldots

C_{66}^* = 0.11
C_{22}^* = 0.32
C_{11}^* = 0.32

[C_{0/\pm45}]

C_{66}^* = 0.14
C_{22}^* = 0.18
C_{11}^* = 0.41

Components: 3D Universal Laminates

3D Trace, GPa
Components: 3D Universal Laminates

C_{11} = C_{22} = 0.405
In-plane: 0.86  Out-plane: 0.14

C_{33}^{*} = 0.057
C_{55}^{*} = 0.024
C_{44}^{*} = 0.017

C_{44}^{*} = 0.020
C_{55}^{*} = 0.020
C_{33}^{*} = 0.057

C_{44}^{*} = 0.016
C_{55}^{*} = 0.025
C_{33}^{*} = 0.057

3D Trace, GPa

C_{11}^{*} = 0.58
C_{22}^{*} = 0.16
C_{66}^{*} = 0.059

[0/±45/90]  [0/±45/90]  [0/±45/90]

C_{11}^{*} = 0.27
C_{22}^{*} = 0.27
C_{66}^{*} = 0.16

C_{22}^{*} = 0.083
C_{11}^{*} = 0.604
C_{66}^{*} = 0.087
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
Unique NCF at Chomarat

Shallow angles, wide range ply thicknesses with high quality from tow spreading, noninvasive stitching, hybrid, ...

\[0/25\]

Flip over

\[-25/0\]

\[20 \leq \phi \leq 30\]
Wide-range GSM to Meet Requirement

Chomarat C-Ply capabilities

Carbon Spread Tows

Optimum GSM/Ply based on 32 plies

Laminate thickness in mil

Laminate thickness of thinnest section mm
Thick-thin Combinations of C-Ply

Thin [0]; thick [\varphi]

Thin [0]; thin [\varphi]

Thick [0]; thin [\varphi]

Black Alum

C-Ply laminates

[0]

[0/\pm45/90]

[0/\pm\varphi]

[0/\pm\varphi/90]

[0/\pm\varphi/\pm\psi/90]

1-axis

4-axis

1-axis

2-axis

2-axis
Layup: 4-axis $[0]_{thk}$ vs 2-axis $[0/45]_{thn/thk}$

Jim Hecht of MAG

More exact estimate:
2.7X faster for thin ply;
5.4X for thick ply

Layup time:
8’ for thick $[0]$
4’ for thin $[0/45]$
2’ for thk $[0/45]$

4-axis Unitape layup

2-axis $[0/45]$ thin-ply layup

Laminate length, m

Time, min
# High Speed Layup of Thick-thin C-Ply

<table>
<thead>
<tr>
<th>Starting C-Ply</th>
<th>1-axis layup 1:0</th>
<th>2-axis layup 2:1</th>
<th>2-axis layup 1:1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin-Thick</td>
<td>High shear</td>
<td>Regular</td>
<td>Low shear</td>
</tr>
<tr>
<td>(33/67/0) – 150 gsm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0/±φ]₂</td>
<td>[0/±φ]₂</td>
<td>[0₂/±φ]</td>
<td></td>
</tr>
<tr>
<td>(33/67/0)</td>
<td>= [π/3]₂ for φ = 60</td>
<td>(50/50/0)</td>
<td></td>
</tr>
<tr>
<td>±ψ/90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5X</td>
<td>(22/67/11)</td>
<td>(33/50/17)</td>
<td>(44/33/22)</td>
</tr>
<tr>
<td>Thin-Thin</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(50/50/0) – 150 gsm</td>
<td>[0₂/±φ]</td>
<td>(50/50/0)</td>
<td></td>
</tr>
<tr>
<td>±ψ/90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>±ψ/90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3X</td>
<td>(33/50/17)</td>
<td>(44/33/22)</td>
<td>(33/33/33)</td>
</tr>
<tr>
<td>Thick-Thin</td>
<td>Low shear</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(67/33/0) – 150 gsm</td>
<td></td>
<td>[0₄/±φ]</td>
<td></td>
</tr>
<tr>
<td>±ψ/90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>±ψ/90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2X</td>
<td>(67/33/0)</td>
<td>(67/33/0)</td>
<td>(33/33/33)</td>
</tr>
</tbody>
</table>
Universal Stiffness Components: C-Ply

\[ [0/\pm \varphi]_2 \]

\[ [0/\varphi_2]; [0/-\varphi_2] \]

Thin-thick
High shear

\[ [0_2/\pm \varphi] \]

\[ [0_2/\varphi]; [0_2/-\varphi] \]

Thin-thin
Regular

\[ [0_4/\pm \varphi] \]

\[ [0_4/\varphi]; [0_4/-\varphi] \]

Thick-thin
Low shear

Universal laminate stiffness

Universal Stiffness Components: C-Ply

\[ [0/\pm 30] \]

\[ [0/\pm 45] \]

\[ [\pi/6] \]

\[ [\pi/3] = 0.34 \]

\[ [\pi/4] \]

\[ [\pi/3] = 0.13 \]
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
Black Aluminum Design: Bricky

Cannot be optimized; manufacturing nightmare

\[
\begin{align*}
[0/\pm45_3] & \quad \begin{aligned}
E_1^* &= 0.45 \\
E_6^* &= 0.14
\end{aligned} \\
(45/55/0) & \quad 0.466 \\
& \quad 0.138
\end{align*}
\]

\[
\begin{align*}
[0/\pm45_2] & \quad \begin{aligned}
E_1^* &= 0.22 \\
E_6^* &= 0.20
\end{aligned} \\
(20/80/0) & \quad 0.268 \\
& \quad 0.187
\end{align*}
\]

\[
\begin{align*}
[0/\pm45_2/90] & \quad \begin{aligned}
E_1^* &= 0.28 \\
E_6^* &= 0.16
\end{aligned} \\
(17/66/17) & \quad 0.277 \\
& \quad 0.161
\end{align*}
\]
C-Ply: Black Aluminum Replacement

\[ [0/\pm \varphi]_2 \]
\[ [0/\varphi_2]; [0/-\varphi_2] \]

Thin-thick
High shear

Trace-normalized stiffness

Off-axis angle

E1*

E6*

Layup ratio

1:0
2:1
1:1

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

Off-axis angle

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

0.22
0.20
0.16
0.28
0.45
0.14

[0/\pm 45_2]

[0/\pm 45_2/90]

[0/\pm 45]
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
Evoluation: Unit Circle Failure Envelope

T700/2510; $E_m^* = 0.15$: Fiber dominated omni envelope

One unit circle in strain for all CFRP, all ply angles

Unit circle in stress for IM7/977-3 $[\pi/4]$ only
Safety Factor R; Failure Index k

\[ R = \text{Strength ratio: } \frac{G_{ij}e_i e_j}{G_i e_i} R^2 + \frac{G_i e_i}{R} = a R^2 + b R = 1 \]  

**Scalable**

K = FEA index:

\[ G_{ij}e_i e_j + G_i e_i = K^2 \]  

**Go or no go only, not scalable**

k = Failure index:

\[ \left[ G_{ij}e_i e_j \right] \left[ \frac{1}{k} \right]^2 + \left[ G_i e_i \right] \left[ \frac{1}{k} \right] = a \left[ \frac{1}{k} \right]^2 + b \left[ \frac{1}{k} \right] = 1 \]  

**Scalable**
Universal Stress-Strain Curves for CFRP

<table>
<thead>
<tr>
<th>Laminate</th>
<th>Strain</th>
<th>Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>T700/2510</td>
<td>0.79</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>-1.10</td>
<td>-0.35</td>
</tr>
<tr>
<td>Soft1</td>
<td>0.69</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>-1.02</td>
<td>-0.23</td>
</tr>
<tr>
<td>Hard1</td>
<td>1.07</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>-1.00</td>
<td>-0.47</td>
</tr>
<tr>
<td>T650/epoxy</td>
<td>1.01</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>-1.00</td>
<td>-0.33</td>
</tr>
<tr>
<td>Hard1</td>
<td>1.07</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>-1.00</td>
<td>-0.47</td>
</tr>
<tr>
<td>IM7/8552</td>
<td>1.01</td>
<td>0.33</td>
</tr>
<tr>
<td></td>
<td>-1.00</td>
<td>-0.33</td>
</tr>
<tr>
<td>Soft1</td>
<td>1.07</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>-1.00</td>
<td>-0.47</td>
</tr>
<tr>
<td>Hard1</td>
<td>0.96</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>-0.91</td>
<td>-0.43</td>
</tr>
<tr>
<td>T4807/MR61</td>
<td>0.82</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>-0.95</td>
<td>-0.27</td>
</tr>
<tr>
<td>Soft2</td>
<td>0.64</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>-1.01</td>
<td>-0.18</td>
</tr>
<tr>
<td>Hard2</td>
<td>0.94</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>-0.74</td>
<td>-0.36</td>
</tr>
<tr>
<td>IM7/MTM45</td>
<td>1.07</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>-1.13</td>
<td>-0.35</td>
</tr>
<tr>
<td>Hard1</td>
<td>1.01</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>-1.26</td>
<td>-0.24</td>
</tr>
<tr>
<td>Median+</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>Median-</td>
<td>-1.00</td>
<td></td>
</tr>
</tbody>
</table>

Metal*: Specific stiffness of Al, Ti and Fe (normalized by density)
Comparison Unit Circle vs WWFE Data
Ply-by-Ply vs Homogenized Plate

Ply-by-ply $R^{(i)}$ of a laminated anisotropic or orthotropic plate

$R = \text{strength ratio} = \text{safety factor}$

Homogeneous anisotropic plate: one $R$

$E_1^\circ = 1/a_{11}^\ast, E_2^\circ = 1/a_{22}^\ast, \ldots$ from trace

Unit circle: $e_x$ and $e_x'$

$R^\text{FPF} \leftrightarrow R^{(i)}$
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
Direct Weight and Layup Rate Method

- Universal virtual laminates
- Boundary-value problem
- Unit circle: failure index $k^{(p)}$
- Tapered thickness: $k^{(p)}_{\text{max}}$
- Layup/zone selection
- Stiffness correction
- Weight/Layup rate
- Material selection

Essential ply properties

<table>
<thead>
<tr>
<th>15 Ply material</th>
<th>Trace</th>
<th>$e_x$</th>
<th>$e_x'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IM6/epoxy</td>
<td>232</td>
<td>17.2</td>
<td>7.6</td>
</tr>
<tr>
<td>IM7/977-3</td>
<td>218</td>
<td>17.0</td>
<td>8.4</td>
</tr>
<tr>
<td>T300/5208</td>
<td>206</td>
<td>8.4</td>
<td>8.4</td>
</tr>
<tr>
<td>IM7/MTM45</td>
<td>195</td>
<td>14.3</td>
<td>9.7</td>
</tr>
<tr>
<td>T800/Cytec</td>
<td>183</td>
<td>23.3</td>
<td>10.2</td>
</tr>
<tr>
<td>IM7/8552</td>
<td>180</td>
<td>15.7</td>
<td>10.7</td>
</tr>
<tr>
<td>T800S/3900</td>
<td>168</td>
<td>19.9</td>
<td>16.6</td>
</tr>
<tr>
<td>T300/F934</td>
<td>168</td>
<td>8.9</td>
<td>8.2</td>
</tr>
<tr>
<td>T700 C-Ply 64</td>
<td>163</td>
<td>20.9</td>
<td>14.1</td>
</tr>
<tr>
<td>AS4/H3501</td>
<td>162</td>
<td>10.5</td>
<td>10.5</td>
</tr>
<tr>
<td>T650/epoxy</td>
<td>160</td>
<td>15.8</td>
<td>11.9</td>
</tr>
<tr>
<td>T4708/MR60H</td>
<td>158</td>
<td>17.8</td>
<td>12.0</td>
</tr>
<tr>
<td>T700/2510</td>
<td>144</td>
<td>17.2</td>
<td>11.5</td>
</tr>
<tr>
<td>AS4/MTM45</td>
<td>143</td>
<td>14.7</td>
<td>11.0</td>
</tr>
<tr>
<td>T700 C-Ply 55</td>
<td>139</td>
<td>20.9</td>
<td>13.8</td>
</tr>
</tbody>
</table>
**k-based Taper**

T700 C-Ply 64 [0/±45/0] laminate

---

**Graph:**

Beam tapering based on unit circle failure index.

- Step 1: constant thickness.
- Step 2: thickness adjusted for stiffness.
- Step 3: thickness scaled to $k$.

**Graph:**

Unit circle failure index - tapered plate.

- Step 2: thickness scaled to $k$.
- Step 3: thickness adjusted for stiffness.

---

**Axes:**

- Thickness variation (mm)
- K variation
- Axial coordinate (mm)
Examples of 1- and 2-axis Layup

1-axis **1:0** layup: Homogenized, single ply drop

2-axis **2:1** layup: Homogenized, single ply drop

2-axis **1:1** layup: Homogenized, single ply drop
Examples of Transition between Layups

Transition from 2:2 to 2:0; 50% axial

Transition from 2:1 to 2:0; 67% axial

Transition from 3:3 to 4:2; 50% axial

Ply sequence

[0₂/90] [0]
[0/90] [0]
[0/90/0₂/90₂] [0₂/90]
Cases Investigated

- **[0/±φ]_2**
  - Thin-thick
  - High shear
- **[0_2/±φ]**
  - Thin-thin
  - Regular
- **[0_4/±φ]**
  - Thick-thin
  - Low shear

Universal laminate stiffness

- **E₁**
- **E₆**

Off-axis angle φ

Cases Investigated:

1. **[0/±φ]_2**
   - [0/φ₂]; [0/-φ₂]
   - Thin-thick
   - High shear

2. **[0_2/±φ]**
   - [0/φ]; [0/-φ]
   - Thin-thin
   - Regular

3. **[0_4/±φ]**
   - [0_2/φ]; [0_2/-φ]
   - Thick-thin
   - Low shear

- **E₁**
- **E₆**

Cases Investigated:

1. **[0/±φ]_2**
   - [0/±30]
   - [0/±45]
   - [π/3] = 0.34
   - [π/4]

2. **[0_2/±φ]**
   - [0/±30/0]
   - [0/±45/0]

3. **[0_4/±φ]**
   - [0_2/0]
   - [0_2/-φ]

- **E₁**
- **E₆**

Cases Investigated:

1. **[0/±φ]_2**
   - [0/±30]
   - [0/±45]
   - [π/3] = 0.34
   - [π/4]

2. **[0_2/±φ]**
   - [0/±30/0]
   - [0/±45/0]

3. **[0_4/±φ]**
   - [0_2/0]
   - [0_2/-φ]

- **E₁**
- **E₆**
2- and 3-zone Layup
Weight Reduction and Layup Speed

- Aluminum
- Black Al
- C-Ply

Bar chart showing weight comparisons for different layups and materials.
Cost Reduction: Weight/speed of C-Ply

Ranking insensitive among values of exponents
Distinct regions between black aluminum and C-Ply

<table>
<thead>
<tr>
<th>Layup ranking</th>
<th>Linear</th>
<th>Wt^2</th>
<th>Speed^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-axis [π/3] 1</td>
<td>0.15</td>
<td>0.12</td>
<td>0.031</td>
</tr>
<tr>
<td>60° thk-thn (1:0) 1</td>
<td>0.20</td>
<td>0.20</td>
<td>0.040</td>
</tr>
<tr>
<td>60° thn-thn 3 2:2, 2:1 and</td>
<td>0.21</td>
<td>0.13</td>
<td>0.066</td>
</tr>
<tr>
<td>60° thk-thn (2:1) 1</td>
<td>0.21</td>
<td>0.13</td>
<td>0.069</td>
</tr>
<tr>
<td>60° thk-thn 2 2:2 and 2:1</td>
<td>0.23</td>
<td>0.14</td>
<td>0.090</td>
</tr>
<tr>
<td>30° thk-thn 2 2:2 and 2:1</td>
<td>0.24</td>
<td>0.15</td>
<td>0.091</td>
</tr>
<tr>
<td>45° thk-thn 2 2:2 and 2:1</td>
<td>0.24</td>
<td>0.15</td>
<td>0.094</td>
</tr>
<tr>
<td>30° thn-thn 2 2:2 and 2:1</td>
<td>0.25</td>
<td>0.16</td>
<td>0.097</td>
</tr>
<tr>
<td>45° thn-thn 2 2:2 and 2:1</td>
<td>0.28</td>
<td>0.20</td>
<td>0.107</td>
</tr>
<tr>
<td>30° thn-thk 2 2:2 and 2:1</td>
<td>0.28</td>
<td>0.20</td>
<td>0.108</td>
</tr>
<tr>
<td>45° thn-thk 3 2:2, 2:1 and</td>
<td>0.29</td>
<td>0.26</td>
<td>0.093</td>
</tr>
<tr>
<td>60° thk-thn (1:1) 1</td>
<td>0.32</td>
<td>0.20</td>
<td>0.158</td>
</tr>
<tr>
<td>Black aluminum</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-axis HARD</td>
<td>0.77</td>
<td>0.91</td>
<td>0.499</td>
</tr>
<tr>
<td>4-axis [π/4]</td>
<td>0.77</td>
<td>0.60</td>
<td>0.771</td>
</tr>
<tr>
<td>4-axis SOFT</td>
<td>1.55</td>
<td>1.84</td>
<td>2.010</td>
</tr>
</tbody>
</table>
Material Selection for Weight Savings

- **Stiffness-controlled**
- **Strength-controlled**
- Built-in safety factor due to stiffness scaling
Topics

• Why trace? The one and only unifying constant
• Theory of trace – simple; easy to understand
• C-Ply by Stanford/Chomarat – bi-angle NCF
• Homogenization – optimizable; high-speed ATL
• Unit circle failure criterion
• Trace-based sizing method – power of C-Ply
• Conclusions
Traditional vs Trace-based Pyramids

Traditional building blocks

Trace-based: just like metals

Coupons from pristine laminates

Composite components

As-designed: pristine

As-built: with defects

Need one and only one universal ply data from [0] for each CFRP

Reduce coupons to one [0] and uniaxial tests
Unleash choices of laminates (not just 4), and
Include processing defects in as-built coupons
Black Aluminum vs Trace: 15 CFRP

\[ E_x = 0.88 \text{ Trace} \]

Black aluminum vs Trace

So much simpler: 4:1 5:2 15:1 15:1 15:1
Relative Cost of Hybrid Laminates

Linear correlation with trace, but not with $E_x$ or others

$$\text{Trace}^{\text{Hybrid}} = v_1 \text{Trace}_1 + v_2 \text{Trace}_2 + \ldots$$
Online, Live Composites Design Workshop XI

February 1-5, 2016; noon to 4 PM PST; 20 hours + homework
US$1,200 including hardcover and e-books, composites app
MS Excel-based MicMac’s and other practical design tools
All sessions recorded/downloadable for individual viewing
Widely recognized as the best online training; no travel
Optional official transcript of 3 credit CE hours for extra fee
Must-learn trace that has revolutionized composites testing

For info/registration: [http://compositesdesign.stanford.edu](http://compositesdesign.stanford.edu)