AA599: Geometric Methods for Nonlinear Control Systems
Homework # 6
Due: Tuesday May 18, 5:00pm

All problems have equal value. Please show all work, not just final answers.

1. Consider the following nonlinear system (a model of a permanent magnet stepper motor)

$$\begin{aligned}
\dot{x}_1 &= -K_1 x_1 + K_2 x_3 \sin(K_5 x_4) + u_1 \\
\dot{x}_2 &= -K_1 x_2 + K_2 x_3 \cos(K_5 x_4) + u_2 \\
\dot{x}_3 &= -K_3 x_1 \sin(K_5 x_4) + K_3 x_2 \cos(K_5 x_4) - K_4 x_3 + K_6 \sin(4K_5 x_4) - \tau_L / J \\
\dot{x}_4 &= x_3
\end{aligned}$$

(Here x_1, x_2 denote currents, x_3 denotes the rotor speed, x_4 is the motor position, J is the rotor inertia, and τ_L is the load torque, which is assumed to be measurable.)

(a) Verify the conditions for feedback linearizability of the system at the point $x_1 = x_2 = x_3 = x_4 = 0$.

(b) Show that the coordinate transformation involved in the linearizing transformation is given as

$$\begin{aligned}
z_1 &= x_4 / K_3 \\
z_2 &= x_3 / K_3 \\
z_3 &= -x_1 \sin(K_4 x_4) + x_2 \cos(K_5 x_4) - K_4 x_3 / K_3 + (K_6 / K_3) \sin(4K_5 x_4) - \tau_L / (J K_3) \\
z_4 &= x_1 \cos(K_5 x_4) + x_2 \sin(K_5 x_4)
\end{aligned}$$

and compute the corresponding linearizing feedback $u = \alpha(x) + \beta(x)v$.

2. Consider the dynamics of a rocket outside the atmosphere as shown in Fig. **??**. The forces which act on the rocket are the gravitational force and the force as delivered by the rocket motor. The control variable is the angle α expressing the direction of the force as delivered

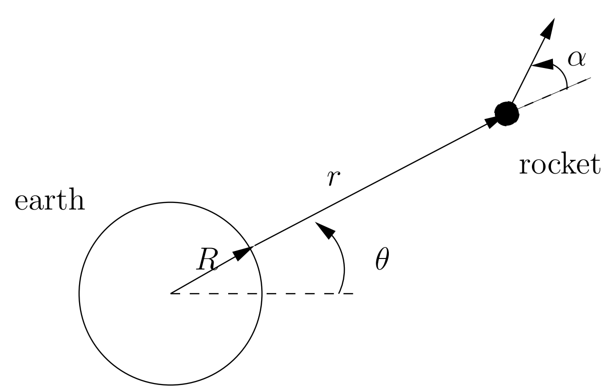

Figure 1: Rocket outside the atmosphere.

by the rocket motor. The dynamics can be written as

$$\begin{aligned}\dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= -gR^2/x_1^2 + \frac{T}{m}\cos(u) + x_1 x_4^2 \\ \dot{x}_4 &= -2x_3x_4/x_1 + \frac{T}{mx_1}\sin(u)\end{aligned}$$

with m the mass of the rocket, g the gravitational constant and R the radius of the earth. Rewrite the system equations as an affine control system using the additional state equation $\dot{u} = w$. Is this system feedback linearizable?

3. Consider the control system

$$\dot{x}(t) = f(x(t)) + u_1(t)g(x(t)) + u_2(t)g_2(x(t))$$

with $x(t)$ taking on values in $\mathbb{R}^3$ and

$$f(x) = \begin{bmatrix} \sin(x_1) \\ x_2 \\ \sinh(x_1 x_2) \end{bmatrix}, \; g_1(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; g_2(x) = \begin{bmatrix} 0 \\ 1 + x_1^2 \\ 1 \end{bmatrix}$$

Does there exist a feedback control law and a change of coordinates such that the resulting system is linear?

4. The vector field in $\mathbb{R}^2$ defined by $[x_1 \partial/\partial x_1 + \cos(x_2)\partial/\partial x_2]$ is nonzero in a neighborhood of 0. Find the change of coordinates that makes it constant in a neighborhood of 0.