AA599: Geometric Methods for Nonlinear Control Systems
Homework # 5
Due: Tuesday May 4, 5:00pm

All problems have equal value. Please show all work, not just final answers.

1. Consider a frictionless, rigid two-link robot manipulator (or double pendulum) with control torques u_1 and u_2 applied at the joints (see Fig. 1). The equations of motion for this system

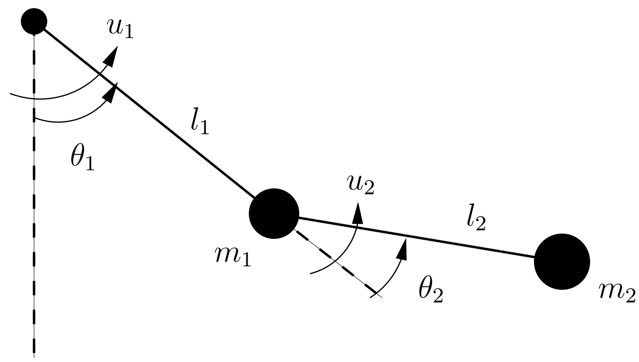

Figure 1: Two-link robot manipulator.

are given by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + k(\theta) = u$$

where

$$\begin{aligned} M(\theta) &= \begin{bmatrix} m_1 l_1^2 + m_2 l_1^2 + m_2 l_2^2 + 2 m_2 l_1 l_2 \cos(\theta_2) & m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2) \\ m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2) & m_2 l_2^2 \end{bmatrix} \\ C(\theta, \dot{\theta}) &= \begin{bmatrix} -m_2 l_1 l_2 \ \sin(\theta_2)\ \dot{\theta}_2\ (2\dot{\theta}_1 + \dot{\theta}_2) \\ m_2 l_1 l_2 \ \sin(\theta_2)\ \dot{\theta}_1^2 \end{bmatrix} \\ k(\theta) &= -\begin{bmatrix} m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) + m_2 g l_2 \sin(\theta_1 + \theta_2) \\ m_2 g l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} \end{aligned}$$

The determinant of M is positive for all θ and therefore the equations can be rewritten as

$$\ddot{\theta} = -M(\theta)^{-1} C(\theta, \dot{\theta}) - M(\theta)^{-1} k(\theta) + M(\theta)^{-1} u.$$

Let the output for this system be the angle of the first joint:

$$y = \theta_1.$$

Linearize this system about $\theta_1 = \theta_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0$ and $u_1 = u_2 = 0$. Show that the linearized system is observable for $g \neq 0$, while it is not observable for $g = 0$. On the other hand, show that $\dim d\mathcal{O} = 4$ even in the case $g = 0$.

2. Is the system from problem 3 of homework 3 observable if U is zero and one observes the trace of A?

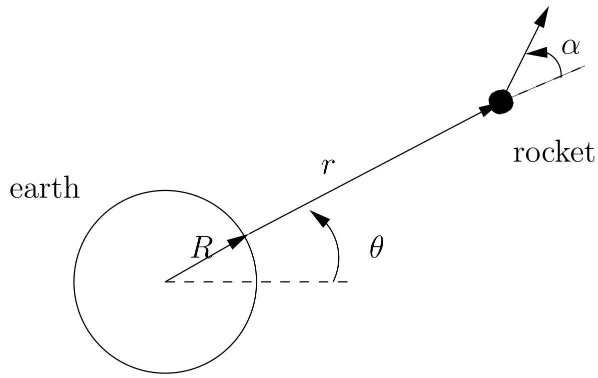

Figure 2: Rocket outside the atmosphere.

3. Consider the dynamics of a rocket outside the atmosphere as shown in Fig. 2. The forces which act on the rocket are the gravitational force and the force as delivered by the rocket motor. The control variable is the angle α expressing the direction of the force as delivered by the rocket motor. The dynamics can be written as

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= -gR^2/x_1^2 + \frac{T}{m}\cos(u) + x_1 x_4^2 \\ \dot{x}_4 &= -2x_3x_4/x_1 + \frac{T}{mx_1}\sin(u) \end{aligned}$$

   with m the mass of the rocket, g the gravitational constant and R the radius of the earth. Rewrite the system equations as an affine control system using the additional state equation $\dot{u} = w$. Is this system feedback linearizable?

4. Consider the control system

$$\dot{x}(t) = f(x(t)) + u_1(t)g(x(t)) + u_2(t)g_2(x(t))$$

   with $x(t)$ taking on values in $\mathbb{R}^3$ and

$$f(x) = \begin{bmatrix} \sin(x_1) \\ x_2 \\ \sinh(x_1x_2) \end{bmatrix}, \; g_1(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; g_2(x) = \begin{bmatrix} 0 \\ 1 + x_1^2 \\ 1 \end{bmatrix}$$

   Does there exist a feedback control law and a change of coordinates such that the resulting system is linear?

5. The vector field in $\mathbb{R}^2$ defined by $[x_1\partial/\partial x_1 + \cos(x_2)\partial/\partial x_2]$ is nonzero in a neighborhood of 0. Find the change of coordinates that makes it constant in a neighborhood of 0.

6. Find an example of a system of the form

$$\dot{x}(t) = f(x(t)) + \sum_{i=1}^{m} g_i(x)u_i(t)$$

that is not linearizable but that is both controllable and linearizable if some of the u's are ignored.