AA599: Geometric Methods for Nonlinear Control Systems
Homework # 4
Due: Tuesday April 27, 5:00pm

All problems have equal value. Please show all work, not just final answers.

1. Consider a system defined on a $n(n+1)/2$-dimensional vector space, realized as pairs (x, Z) where x is an n-vector and $Z = -Z^T$ is a skew-symmetric matrix. The equations of motion are

$$\begin{aligned} \dot{x} &= u \\ \dot{Z} &= xu^T - ux^T \end{aligned}$$

   Show that this system is controllable.

2. Consider a frictionless, rigid two-link robot manipulator (or double pendulum) with control torques u_1 and u_2 applied at the joints (see Fig. 1). The equations of motion for this system

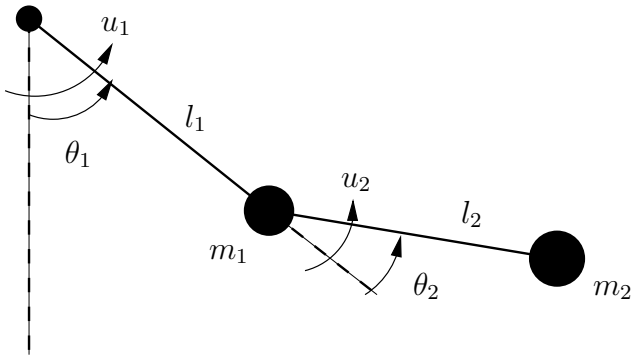

Figure 1: Two-link robot manipulator.

   are given by

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + k(\theta) = u$$

   where

$$\begin{aligned} M(\theta) &= \begin{bmatrix} m_1 l_1^2 + m_2 l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(\theta_2) & m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2) \\ m_2 l_2^2 + m_2 l_1 l_2 \cos(\theta_2) & m_2 l_2^2 \end{bmatrix} \\ C(\theta, \dot{\theta}) &= \begin{bmatrix} -m_2 l_1 l_2 \ \sin(\theta_2)\ \dot{\theta}_2 \ (2\dot{\theta}_1 + \dot{\theta}_2) \\ m_2 l_1 l_2 \ \sin(\theta_2)\ \dot{\theta}_1^2 \end{bmatrix} \\ k(\theta) &= -\begin{bmatrix} m_1 g l_1 \sin(\theta_1) + m_2 g l_1 \sin(\theta_1) + m_2 g l_2 \sin(\theta_1 + \theta_2) \\ m_2 g l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} \end{aligned}$$

   The determinant of M is positive for all θ and therefore the equations can be rewritten as

$$\ddot{\theta} = -M(\theta)^{-1} C(\theta, \dot{\theta}) - M(\theta)^{-1} k(\theta) + M(\theta)^{-1} u.$$

   Let the output for this system be the angle of the first joint:

$$y = \theta_1.$$

Linearize this system about $\theta_1 = \theta_2 = \dot{\theta}_1 = \dot{\theta}_2 = 0$ and $u_1 = u_2 = 0$. Show that the linearized system is observable for $g \neq 0$, while it is not observable for $g = 0$. On the other hand, show that $\dim d\mathcal{O} = 4$ even in the case $g = 0$.

3. Please write a 1 page abstract for your project topic.