AA461 Advanced Propulsion
Autumn 2007

HOMEWORK #5

DUE: THURSDAY, NOVEMBER 1, 4 pm

1. (6 pts.) Problem 8-1 from *Hill and Peterson*.

2. (10 pts) Consider the first stage of axial turbine with zero degree of reaction. At an off-design point, the shaft speed drops by 20%, but the mass flow is unchanged from the design point value.

   a) Draw, on one figure, the velocity diagrams for the design and off-design conditions upstream and downstream of the rotor stage. Denote the design conditions with an asterix (*) and label the conditions upstream of the rotor as "1"; for conditions downstream of the rotor, "2".

   b) Determine the percentage change in shaft output power from the stage in going from the design condition to the off-design condition.

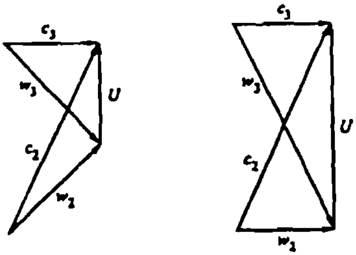
3. (8 pts.) Problem 8-7 from *Hill and Peterson*.

Please note that the FIRST EXAM will be Friday, November 2, in class.

Solutions to this HW set will be posted Thursday, November 1, at 5 pm

(end)

8-1. The figure shows velocity diagrams for impulse and 50% reaction turbines at different rotor speeds. The subscripts 2 and 3 denote conditions before and after the rotor, respectively. In both cases the absolute velocity c_2 is 400 m/s at an angle $\alpha_2 = 70°$, and the absolute exhaust velocity c_3 is axial. If the blades are uncooled and well insulated from the turbine disc, approximately what would the equilibrium temperature of the blades be in each case, for an inlet gas temperature $T_{02} = 1100$ K?



Take $r_f = 0.9$ $\quad C_p = 1.12$ kJ/kg.K

R=0.5

$T_b \propto T_{o\,rel}$

$\propto T_2 + r_f \dfrac{w_2^2}{2C_p}$

$$T_b \propto T_{01} - \frac{C_2^2}{2C_p} + r_f \frac{w_2^2}{2C_p}$$

$w_2 = 400 \cos 70°$
$= 136.8$ m/s

$$T_b = 1100 - \frac{400^2}{2(1120)} + \frac{0.9(136.8)^2}{2(1120)}$$
$= 1100 - 71.4 + 7.6$
$= \underline{\underline{1036}}$ K

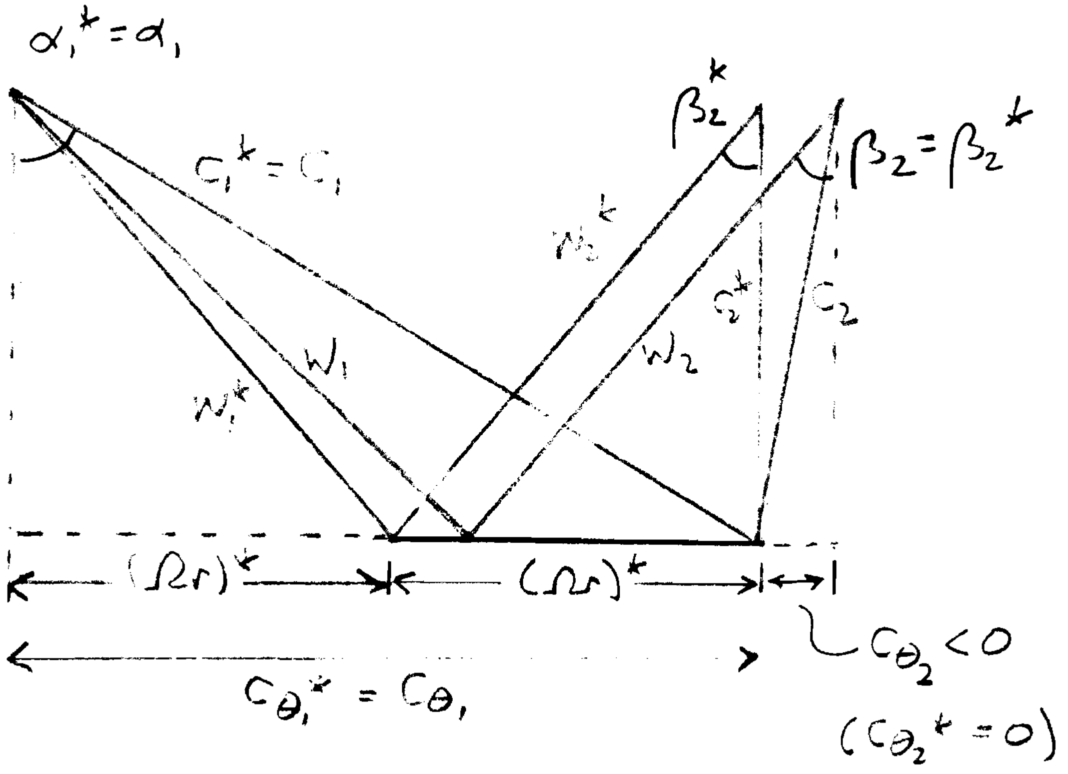
R=0

$w_2 = \sqrt{(400\cos 70°)^2 + \left(\frac{400 \sin 70°}{2}\right)^2}$
$= 234.2$ m/s

$$T_b = 1100 - \frac{400^2}{2(1120)} + \frac{0.9(232.4)^2}{2(1120)}$$
$= 1100 - 71.4 + 21.7$
$= \underline{\underline{1050}}$ K

① Axial turbine w/ zero degree of reaction → $R = 0$ ⇒ impulse turbine stage

a) velocity diagrams → $*$ = design value



Notes:

- rotor inflow conditions unchanged ⇒ $c_1 = c_1^*$, $c_{\theta_1} = c_{\theta_1}^*$ $c_{z_1} = c_{z_1}^*$
- for design condition, impulse turbine ⇒ $c_2 = c_{z_2} = c_{z_1}^*$ ⇒ $c_{\theta_2}^* = 0$, $c_{\theta_1}^* = 2(\Omega r)^*$
- all axial components (c_{z_1}, c_z', w_z, ... etc) are identical

now blade speed $= \Omega r < (\Omega r)^*$
C_z's, W_z's unchanged

consequences:
1) $W_1 > W_1^*$ → lower blade speed means W_1 shifts closer to C_1
($W_1 = C_1$ for $\Omega r = 0$)

2) assuming blade stall does not occur, expect no change in β_2, i.e. $\beta_2 = \beta_2^*$

3) the offset between the vectors W_2 and C_2 is (just like at ①) the quantity Ωr

b) now calculate the change in output power. The mass flow rate is unchanged, so

$$\frac{\dot{W}}{\dot{W}^*} = \frac{\dot{m}\,\Delta h_0}{\dot{m}\,\Delta h_0^*} = \frac{\Delta h_0}{\Delta h_0^*} = \frac{\Omega r\left[C_{\theta_1} - C_{\theta_2}\right]}{\Omega^* r\left[C_{\theta_1}^* - C_{\theta_2}^*\right]}$$

For design condition impulse turbine (note $r^* = r$)
$$\Rightarrow \Delta h_0 = \Omega^* r\left[C_{\theta_1}^* - C_{\theta_2}^*\right]$$
$$= \Omega^* r\left[2(\Omega r)^* - 0\right] = 2(\Omega^* r)^2 \checkmark$$

Now for off-design condition

$$\Delta h_o = \Omega r [c_{\theta_1} - c_{\theta_2}]$$ (note $c_{\theta_2} < 0$ as sketched)

(stagnation pressure change)

$$= \Omega r \left[c_{\theta_1}^{*} - (\Omega r - \Omega r^{*}) \right]$$

$$= \Omega r \left[2\Omega^{*} r - (\Omega r - \Omega^{*} r) \right]$$

$$= \Omega r \left[3\Omega^{*} r - \Omega r \right]$$

but $\Omega r = 0.8\, \Omega^{*} r$ @ off-design point

$$\Rightarrow \Delta h_o = 0.8\, \Omega^{*} r \left[3\Omega^{*} r - 0.8\, \Omega^{*} r \right]$$

$$= (\Omega^{*} r)^2 \left[3(0.8) - 0.8(0.8) \right]$$

$$= 1.76 (\Omega^{*} r)^2$$

$\Rightarrow$ so power reduction $\Rightarrow$

$$\frac{\dot{W}}{\dot{W}^{*}} = \frac{\Delta h_o}{\Delta h_o^{*}} = \frac{1.76 (\Omega^{*} r)^2}{2 (\Omega^{*} r)^2} = \frac{1.76}{2} = \underline{0.88}$$

$\Rightarrow$ 12 % decrease in output power.

8-7. An axial turbine (for one stage of a multistage turbine) is to be designed for a work ratio at the mid-radius of

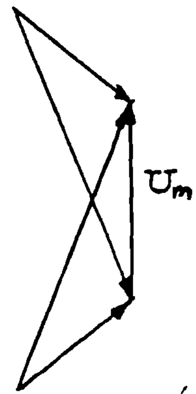
$$\left(\frac{\Delta c_\theta}{U}\right)_m = 2$$

and a free-vortex swirl distribution upstream and downstream of the rotor. At the mid-radius the degree of reaction is to be 50%. The hub–tip ratio is to be 0.8, and the question is whether free-vortex blading would be satisfactory.

At the hub radius the stator exit angle α_{1h} is 70° to the axial direction.

a. Draw the mean and hub velocity triangles (roughly to scale).
b. Determine the axial velocity ratio c_z/U_m
c. Determine the rotor blade angles β_{2h} and β_{3h} at the hub radius.
d. Determine the degree of reaction at the hub radius.

$\left(\frac{\Delta C_\theta}{U}\right)_m = 2 \qquad R_m = 0.5 \qquad \zeta = 0.8$



Mid-Radius

See Fig. 8.8 in H&P; note expect "symmetry" for 50% reaction (e.g. $W_{\theta_2} = -C_{\theta_3}$, etc.)

Four key steps:

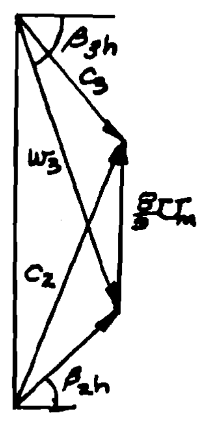
1) note signs, so if $W_{\theta_2} > 0$, $W_{\theta_3} < 0$, etc.
2) note it's "free vortex", so $rC_\theta = \text{const} \rightarrow r_h C_{\theta_h} = r_m C_{\theta_m}$, etc
3) the midpoint °R is given °R $= \frac{1}{2}$
4) from the triangle "geometry" can relate C_θ's, W_θ's, and U @ midpoint

(b) $C_z = C_{\theta 2h}/\tan\alpha_{2h}$

$\quad = C_{\theta 2m}\left(\frac{r_m}{r_h}\right)/\tan\alpha_{2h}$

$\left(\frac{C_z}{U}\right)_m = \left(\frac{C_{\theta 2}}{U}\right)_m \left(\frac{r_m}{r_h}\right)/\tan\alpha_{2h}$

$\quad = 1.5\left(\frac{0.9}{0.8}\right)/\tan 70° = \underline{\underline{0.614}}$



hub

(c) $\tan\beta_{2h} = \dfrac{C_{\theta 2h} - U_h}{C_z}$

$\quad = \tan\alpha_{2h} - \left(\frac{U}{C_z}\right)_m \left(\frac{r_h}{r_m}\right)$

$\quad = \tan 70° - \frac{1}{0.614}\left(\frac{0.8}{0.9}\right)$

$\beta_{2h} = \underline{52.4°}$

$\tan\beta_{3h} = \dfrac{W_{\theta 3h}}{C_z} = \dfrac{C_{\theta 3h} - U_h}{C_z} = \dfrac{C_{\theta 3m}}{C_z}\dfrac{r_m}{r_h} - \left(\frac{U}{C_z}\right)_m \dfrac{r_h}{r_m}$

$\quad = \left(\frac{U}{C_z}\right)_m \left[-\frac{1}{2}\frac{r_m}{r_h} - \frac{r_h}{r_m}\right] = \frac{1}{0.614}\left[\frac{1}{2}\frac{0.9}{0.8} - \frac{0.8}{0.9}\right]$

$\beta_{3h} = \underline{-67.1°}$

(d) $R_h = -\dfrac{W_{\theta 2h} + W_{\theta 3h}}{2U_h} = -\frac{1}{2}\left(\frac{C_z}{U}\right)_h (\tan\beta_{2h} + \tan\beta_{3h})$

$\quad = \frac{1}{2}\left(\frac{C_z}{U}\right)_m \left(\frac{r_m}{r_h}\right)(\tan\beta_{2h} + \tan\beta_{3h})$

$\quad = \frac{1}{2}(0.614)\left(\frac{0.9}{0.8}\right)(\tan 52.4 + \tan(-67.1))$

$\quad = \underline{\underline{-0.368}}$