AA360 Propulsion Spring 2008 **NAME** Solution
QUIZ #21 (7.5% of Final Course Grade)
May 28, 2008

20 minutes. Closed book, closed notes, closed homework and homework solutions.

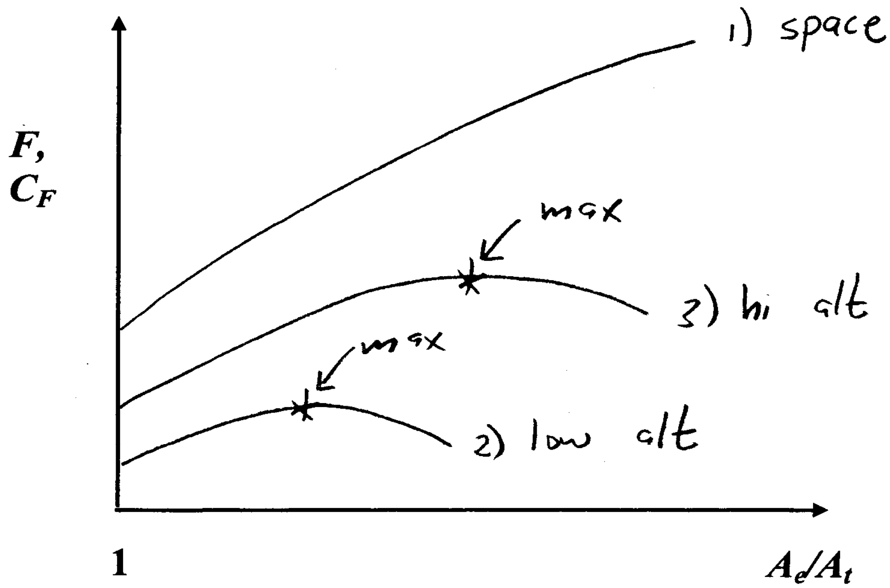
1. (8 pts.) Using the axes provided, sketch qualitatively the variation in thrust (***F***), or thrust coefficient (***C_F***) with the area ratio (***A_e/A_t***) for isentropic rocket nozzles. Assume that all combustion chamber conditions (p_c, T_c, γ, A_t) are constant. Draw and clearly label curves for:

1) space operation, $p_a = 0$;

2) a low-altitude condition where $p_a = p_1$; and

3) an intermediate altitude where $p_a = p_2 < p_1$ (but $p_1 > 0$).

Indicate clearly any/all points of "maximum thrust."



2. (10 pts.) Consider the following three expressions:

i) $f = \dfrac{c_p T_a}{h_{PR}}(\tau_b - 1)$ ii) $f = \dfrac{c_p T_a}{h_{PR}}(\tau_\lambda - \tau_r)$ iii) $f = \dfrac{c_p T_a}{h_{PR}}\sqrt{\dfrac{\tau_\lambda}{\tau_r}}\dfrac{(\gamma - 1)M^2}{\tau_r}$

Identify which of the three is the correct expression for the fuel/air ratio of an ideal ramjet (no losses, γ, R, c_p all constant, and $\dot{m}_f \ll \dot{m}_a$).

Next, present the analytical derivation (use the next page, and symbols, not words!) that confirms your choice above.

Note: Use the ramjet numbering convention:

① = diffuser inlet, ②=③ = diffuser exit, ④=⑤ = combustor exit, ⑥ = nozzle exit.

(continued on back)

2. continued (space for *simple* derivation)

$$\dot{m}_f h_{PR} = c_p \dot{m}_a (T_{t4} - T_{t3}) \; ; \; T_{t3} = T_{t2}$$

$$\Rightarrow f = \frac{\dot{m}_f}{\dot{m}_a} = \frac{c_p T_a}{h_{pr}} \left(\frac{T_{t4}}{T_a} - \frac{T_{t2}}{T_a} \right)$$

$$\frac{T_{t4}}{T_a} = \tau_\lambda \qquad \frac{T_{t2}}{T_a} = \tau_r$$

3. (18 pts) The chamber pressure in a particular rocket motor is increased. The nozzle flow remains fully supersonic and isentropic at all times, and the combustion chamber temperature, T_c, the nozzle throat area, A_t, nozzle area ratio A_e/A_t and the gas properties (R, γ) are all unchanged. The ambient is at a constant pressure, p_a (e.g., a missile application).

Indicate, *with brief explanations for each item*, how the following performance quantities change due to the increase in chamber pressure.

For full credit both the answer and the explanation must be correct!

a) the mass flow at the nozzle exit, $\dot{m}_e$ — **(increases)** — decreases — stays the same

Explanation: $P_c \uparrow$ but A_t, T_c, properties = const
or $\Rightarrow \dot{m} = [P_c A_t / \sqrt{R T_c}]\,\Gamma(\gamma)$

b) the gas velocity at the nozzle exit, u_e — increases — decreases — **(stays the same)**

Explanation: same A_e/A_t → same M_e
same T_c, M_e ⇒ same a_e
⇒ same u_e

c) the nozzle exit static pressure, p_e, — **(increases)** — decreases — stays the same

Explanation: as b) same M_e → same P_e/P_c
$P_c \uparrow \Rightarrow P_e \uparrow$

d) the specific impulse, I_{sp} — **(increases)** — decreases — stays the same

Explanation: $I_{sp} = F/\dot{m} g_c$ $\quad F = \dot{m} u_e + (P_e - P_a) A_e$
so $I_{sp} \sim u_e + (P_e - P_a) A_e / \dot{m}$ $\quad u_e = \text{const}$ b)

(end)