Coalescence of multiple plasmoids as a means of efficient spheromak formation

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Abstract We have produced single bursts of helicity from the source in the SSPX spheromak in order to study the efficiency of the simplest example of helicity injection. We find that the helicity injection rate can be written in terms of the injected current and an inductance, and that a simple circuit analogue demonstrates unambiguously the relationship of helicity to energy: helicity injection is the addition of inductive loops. While helicity balances point to the conservation of helicity, the electrical efficiency is around 15%. However, in the expulsion of the loop, electrical energy is converted to directional motion, which may be recoverable usefully as heat, thus the efficiency of the injection process is arguably quite high. Integral to this notion of helicity injection is the idea that reconnection is necessary: without disconnection from the source by a reconnection event, the spheromak fields are just proportional to the injected current. Sometimes the multiple bursts occur spontaneously and cause a step-wise increase in the field (and helicity). However, in all instances when the current remains above the ejection threshold for t>50μs, the n=1 mode initiates and builds field, although with much reduced efficiency, and to a level which is symptomatic of no reconnection ($B_{spheromak} \propto I_{inj}$).

I. Introduction

The success of the spheromak as a confinement concept depends not only on it's ability to hold pressure and confine heat, but also on a concept known as 'current amplification'. Over the short history of the spheromak, beta has been demonstrated to be reasonable (a few percent [1]), and the confinement is seen to improve markedly when the main instabilities are suppressed (also [1]). However, in order to get to reactor-relevant temperatures, the field strength of the spheromak needs to be increased well beyond present achievements. We are addressing current amplification (A_I=I_tor/I_inj) by use of a pulse-forming network PFN that can deliver a current pulse of 250kA for 2ms, and by use of a variable vacuum field configuration. The work presented here is somewhat pedagogic, in that only one of the bias coils is used (the main solenoid), and the formation bank is used in isolation from the PFN. We have been able to generate single bursts of helicity (plasmoids) by bringing the formation bank current just up to the ejection threshold for a short instant (FIGURE 1). The aim of this work was to produce multiple plasmoids under controlled conditions and to establish if the plasmoids would merge and form higher field-strength spheromaks, thereby increasing the current amplification of the spheromak – this was not achieved controllably, but observed to occur spontaneously: below we present plans to survey this phenomenon with greater rigor.

The main assumption on which current amplification by the merging of multiple plasmoids rests is that the plasmoids will merge and produce a spheromak with a helicity content equal to the sum of the individual plasmoids. Such a result has been demonstrated in the laboratory before by a few authors ([2-4]), and so the endeavor seems to have firm foundation. Critical also to the production of the plasmoids is an understanding of the operation of the coaxial source: here a large body of thruster papers

([5] and references therein), and presently ongoing work concerning the propagation of current sheets in magnetized guns [6] contributes physical insight into the 'helicity injector' of the spheromak. Parallels may also be drawn between our work and that of the solar plasma physicist: namely in the production of solar prominences and flares [7].

The SSPX experiment is described in detail in reference [8], but briefly: it is a Marshall-gun-driven spheromak, 1m in diameter with a gun that is of equal radius to the flux conserver, with 9 independently programmable field coils that generate the vacuum field. The plasma is well diagnosed [9], although here we consider only the magnetic- and gun-circuit-diagnostics. We have achieved clean conditions with the burn-through of all impurities (except for OVI), and, when running clean, the most prominent peak in the spectrum is the Lyman-alpha line, and radiated power is <10% of total input power.

The paper is structured as follows. Section II contains the results and anlaysis; section III outlines our plans; and section IV is the conclusion. 'Burst' and 'plasmoid' are used interchangeably in the text – conceptually, a plasmoid is a blob of plasma that is ejected as a discrete burst from the gun.

II. Results and analysis

The conceptual model shown in FIGURE 2 assists in the interpretation of the bursting behavior shown in FIGURE 1 (see numerals on figure 1 also). The obvious sign of a burst is that the gun voltage increases inductively (V=IdL/dt) as the current sheet expands into the flux-conserver (plots 1-2). At some time, the current sheet ceases to expand, and the inductive component of the gun-voltage falls. For a short while before the plasmoid disconnects, the impedance is purely resistive and the field does not increase (no L-dot or I-dot contributions to V_gun). Understanding the origin of the gun-voltage that contributes to the rise in field allows us to write an expression for the helicity injection:

$$\dot{K} = 2V_g\psi_g = 2I_g\psi_g\dot{L}$$

A moment before the burst is released from the gun, a pressure balance needed to be met in which $B_\theta=B_p$. This allows the injector flux to be expressed in terms of the injector current: $B_\theta = \frac{\mu_0 I}{2\pi R}, B_p = \frac{\psi_g}{2\pi R\Delta/2}$ for a sharp boundary model in which the inter-electrode gap width is Δ, and radius, R, one obtains:

$$\psi_g = \mu_0 I_g \Delta / 2$$

Substituting into the expression for the injection rate, we arrive at:

$$\dot{K} = \mu_0 I_g^2 .\Delta L / \tau$$

We choose to write the rate of change of inductance in terms of the ultimate inductance of the loop and a time, τ, as this is somewhat instructive. For the single burst, τ is the time taken for the plasmoid to fully disconnect from the gun, hence entails reconnection. It follows that if the plasmoid does not disconnect, so that another current loop can form, the helicity of the system is defined explicitly in terms of the inductance and the current: in this case, $B_{spheromak}$ will be proportional to I_g.

Helicity injection by bursting can be understood by use of a simple circuit to describe the process. FIGURE 3 shows the circuit: the injected current remains constant over the period of injection, and drives a load with resistance, R and variable inductance, L. The circuit increases in inductance as the current sheet expands, until the plasmoid disconnects, when the 'switches', S, close in both circuits simultaneously. There is no sudden reversal of voltage, as the flux stored in the inductor (plasmoid) cannot leave the flux-conserver. The magnetic energy of the burst is simply the $I^2L/2$, and for a system that is close to the Taylor state, the energy and helicity are related by:

$$\dot{K} = \frac{2\mu_0}{\lambda}\dot{W} = \frac{\mu_0}{\lambda} I_g^{\,2} L / \tau$$

And the expression for the injection rate is re-obtained for $\lambda \sim 1/\Delta$.

The efficiency of the injection process is determined by the ratio of the useful work done to the total work. For the circuit shown in FIGURE 3, the efficiency can be stated as:

$$\begin{aligned} \varepsilon &= \frac{LI^2}{LI^2 + 2RI^2\tau} \\ &= \frac{L}{L + 2R\tau} = \frac{1}{1 + 2\tau/t} \end{aligned}$$

Where t=L/R, and it could be expected that efficiencies approaching unity can be obtained if the time that the plasmoid remains connected to the source is small compared to the L/R time of the plasmoid. This can be shown to be satisfied for the helicity injection process, in which the injection time is short compared to the dissipation time. However, in pushing a plasmoid out of the gun, electrical energy is converted into directional energy of the blob. Some of the directional energy should be recoverable as heat for a sufficiently collisional plasma, but only a fraction of the ions that are accelerated in this manner can be contained. The directional energy is included in the numerator (in the category of useful work) in the expression for the efficiency[1]:

$$\varepsilon = \frac{LI^2 + Mv^2}{LI^2 + Mv^2 + 2RI^2\tau}$$

Efficient spheromak formation becomes an optimization problem: L depends on v, v depends on M, and only an unknown fraction of the directional energy (perhaps some fraction related to beta) can be useful. For short burst times (~10μs), the energy that is consumed by the resistance is a small fraction of the total input (~10%). In the experiment, we measure the total magnetic energy of the spheromak obtained after bursting (with a coil calibrated using the CORSICA code to give magnetic energy), and

[1] The radiated power fraction is low for J/n>1e-14Am, assumed satisfied here. Charge exchange losses are unknown, and ignored.

find that it contains about 15% of the total gun input power (FIGURE 4). We also observe high energy ions: using an ion Doppler spectrometer tuned to an OV line, the mean ion temperature is seen to be around 600eV during initial ejection from the gun. If the bulk of the initial gas puff is accelerated up to these energies, the remaining 75% of gun input power can be readily assigned to this process. (The energies are not inconsistent with the speed of the ejected plasmoid ~1e5m/s)

Multiple bursts are observed in SSPX, although these occur spontaneously. FIGURE 5 shows one clear example of two bursts that increase the measured field strength of the spheromak in an almost step-wise fashion. We operated the source close to the ejection threshold, and attempted to coax the phenomenon to repeat, but without success. We also attempted to produce two bursts by splitting the formation bank into two separate halves and triggering independently – this was unsuccessful for two reasons: 1) the first spheromak would decay away before the 2nd burst could eject; and 2) bringing the 2nd pulse in closer in time to the 1st caused the 2nd current pulse to add to the first and the current was raised significantly above the ejection threshold, which gave rise to the n=1 mode. We plan to be more rigorous in our examination of this phenomenon (see below).

If the current path changes from flowing in the gun to flowing in the flux-conserver, the bursting process will cease and the n=1 mode starts. This cessation of the bursting activity is seen in nearly every shot: initially the bursts occur (previously disregarded as a 'transient' phenomenon), then within a few microseconds, the bursting stops and the n=1 begins. It can be shown (but space does not permit it – see [10]) that the current path changes during this transition: with time, the fluctuations in field cease in the gun and almost all magnetic activity occurs in the flux-conserver. The entry-region current (measured by a toroidal field coil at the mouth of the gun, calibrated to read axial current as a partial Rogowski coil) is seen to increase with time, indicating that the injected current increasingly returns to the wall in the flux-conserver. As these processes occur, the n=1 mode begins to rotate, and the field-strength of the spheromak continues to rise. The peak in field is reached at the peak of injected current. In our initial paragraph of this section we presented the argument that if the plasmoid does not disconnect from the gun, then the field of the spheromak will be explicitly determined by the injected current and the ultimate inductance of the loop. This appears to be the case for spheromaks formed with the n=1 mode: a database of all formation shots shows that the peak field of the spheromak is a rigid function of the injected current ($B_{spheromak}$ [T] = 0.6 I_{inj} [MA]). Furthermore, we can show here that the efficiency of formation with the single bursts tends to be much more efficient than with the n=1 mode alone. FIGURE 6 shows the evolutia of the total input energy and the edge magnetic field of two shots – the solid line, exhibiting strong n=1 mode (consuming 250kJ/T), and the dashed line, exhibiting the bursting behavior (consuming 66kJ/T). Such an efficiency is typical for the database of shots surveyed.

III. Further work

Given the conceptual simplicity of the bursting phenomenon, and almost tangible benefits in terms of current amplification and efficiency, we would like to explore the process under more controllable conditions. To this end, we have performed SPICE modeling of the circuit to produce 6 bursts of equal amplitude and will make modifications to the bank in due course. We have also designed and are installing a magnetic field probe to survey the field structure in the mouth of the injector. This probe will tell us quite unambiguously where the current is flowing in the gun (from toroidal field measurements) and the distribution of the flux (from axial measurements of field).

We are also exploring the limits to current amplification by bursting. Conjecturally, one ought to expect the current amplification to reach a limit when the spheromak has grown to a field-strength that is sufficient to re-establish a force-balance at the mouth of the injector (stuffed gun). The limit in field of the spheromak will then be determined by the maximum field obtainable in the gun, implying that a smaller gun radius may immediately give higher field-strength spheromaks.

IV. Conclusion

We have produced single bursts of helicity from the gun in SSPX. The injection of helicity by this method is tractable: addition of helicity can be understood as the addition of inductive loops. The process is efficient, in that ~15% of the injected energy is transferred to the field, 10% to resistivity, while the remaining 75% can be readily assigned to ion acceleration (of which some fraction is useful). Occasionally, multiple bursts are observed that increase the field in a step-wise fashion, indicating that merging may be taking place. The bursting ceases when the current path changes, and the n=1 mode starts up. Bursting is more efficient at forming field than by action of the n=1 mode by a factor of >3.

Acknowledgements

This work was performed under the auspices of the US DoE by the University of California Lawrence Livermore National Lab. under Contract No. W-7405-ENG-48. Useful conversations with T. K. Fowler, and K. McCollam are acknowledged.

References

[1] H. S. McLean, S. Woodruff *et al* Phys. Rev. Lett. Accepted for publication 2/19/02

[2] M. R. Brown and P. M. Bellan Phys. Rev. Lett. Vol.64, No.18, p2144 (1990)

[3] Y. Ono, A. Morita, M. Katsurai and M. Yamada Phys. Fluids B 5 (10) p3691 (1993)

[4] M. Yamada, Y.Ono, A. Hayakawa, M. Katsurai and F. W. Perkins Phys. Rev. Lett. Vol. 65, No.6 p721 (1990)

[5] Lovberg R H 1971 *Methods in Experimental Physics – Plasma Physics Pt. B* **9** 251

[6] Markusic T and Choueiri E IEPC paper: IEPC-99-206 Presented at the 26th International Electric Propulsion Conference, Kitakyushu, JAPAN, October 17-21,1999. See also http://alfven.princeton.edu/graphics/moviea.mov for a movie of a current sheet that propagates by JxB forces…

[7]

[8] E.B. Hooper, L.D. Pearlstein, R.H. Bulmer, Nuclear Fusion **39**, 863 (1999)

[9]H. S. McLean, et al., Rev. Sci. Instr. **72**, 556, (2001).

[10] S. Woodruff – Poster at this workshop, available at http://www.mfe-science.org/sspx/CTworkshop.pdf

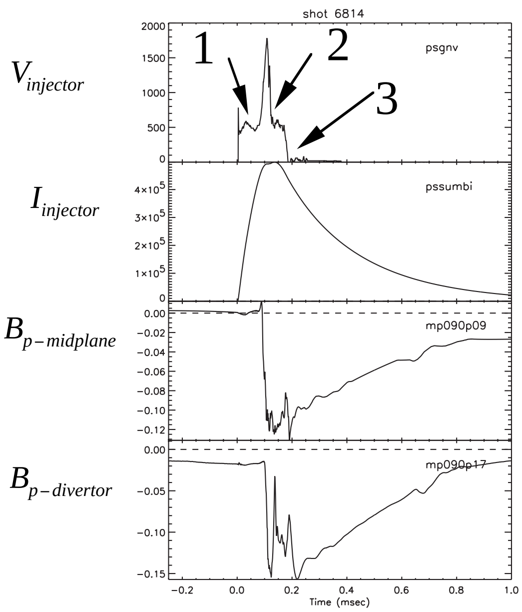

Figure 1. Gun voltage, injector current and edge field for a single 'burst' of helicity

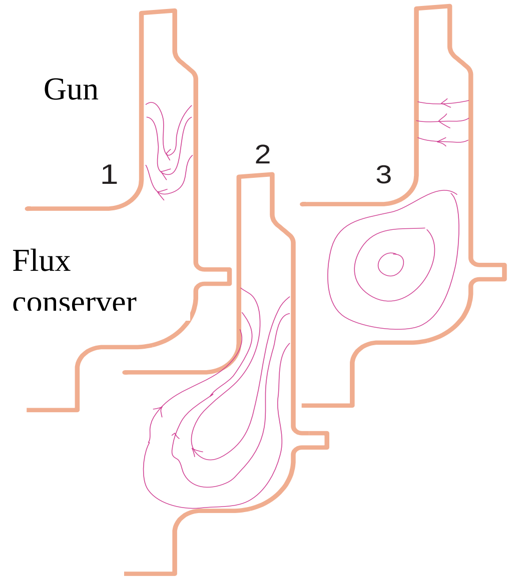

Figure 2. Schematic evolution of the burst – note numbered stages correspond to figure 1.

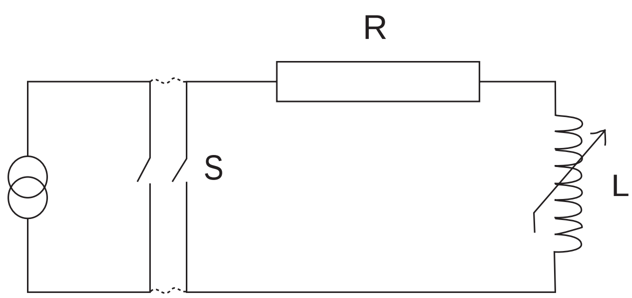

Figure 3. Circuit analogue for helicity injection: a temporary connection breaks, S closes as the plasmoid disconnects

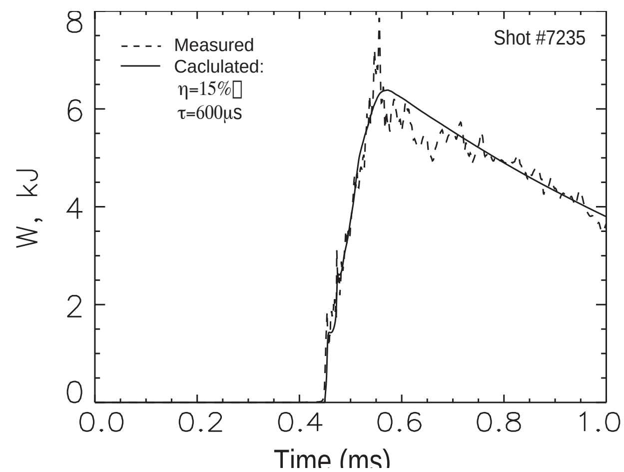

Figure 4. Measured and calculated energy evolutia: nearly 15% of gun power ends up in field.

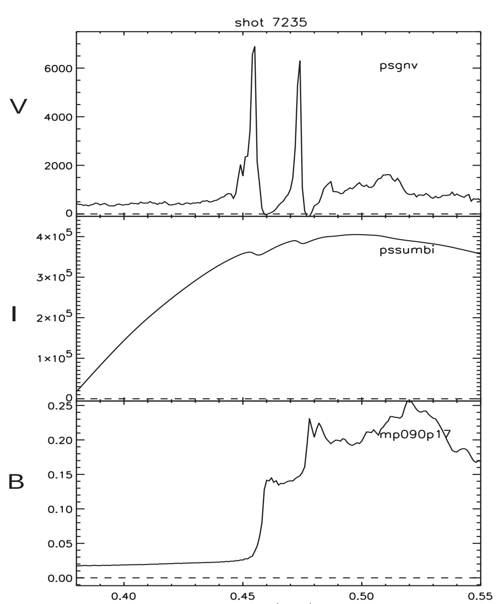

Figure 5. Gun voltage, current, and edge-field for two bursts from the injector that occurred spontaneously and built field

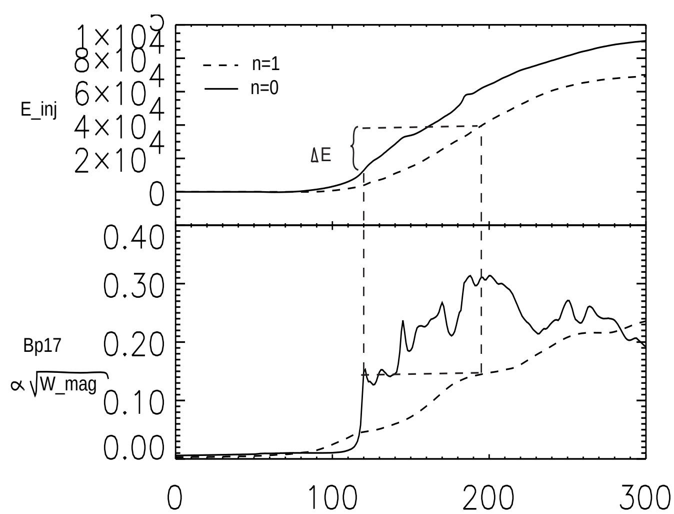

Figure 6. Input energy (top) and edge field (bottom) for two cases: burst (solid-line), and n=1 mode (dashed). Bursting quits after ~140μs