Study of Equilibrium Operation by Rotating Magnetic Field Current Drive in Field-Reversed Configuration

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Abstract

The stability is studied on the balance of the forces exerted on the electrons by the resistive friction and the Rotating Magnetic Field (RMF) applied for the sake of maintaining Field Reversed Configuration (FRC) in steady state. The simple analytical model such as infinite-long plasma, rigidly rotating ions and electrons and uniform plasma density are used. The linear stability analysis of the balance is carried out in the reduced zero-dimensional model which include the effects of ion rotation, radial plasma flow and separatrix radius change due to the flux conservation within the flux conserver. The analytical expression which gives the stability criterion is derived from the eigenvalues of the linearized equations. Based upon the stability criterion, an interpretation of the present experimental results and comments of the future experiments are given as for the penetration of the RMF into the FRC.

1. Introduction

The Field Reversed Configuration (FRC) possesses the advantages of the other candidate of a magnetic confinement fusion in the viewpoints of the high beta, intrinsic diverter configuration and engineering simple structure. However, the lifetime of FRCs produced by the field reversed theta pinch is limited within several hundreds of microseconds due to a lack of mechanism to sustain the current against Joule dissipation. It has been proposed that the Rotating Magnetic Field (RMF) is externally applied in order to keep the reversed configuration in steady state.[1] The experiment has been taken up intendedly to verify that the aim is right.[2] The purpose of the study is to examine analytically the stability of the equilibrium achieved by the balance between a resistive decay of current and a drive of current by the RMF considering the effects of ion rotation as well as separatrix change under the flux conservation.

2. Basic Equations

When we assume the rigid rotation of an electron fluid, the RMF penetrated into an FRC plasma is analytically given by [3]

$$\tilde{B}_r = \frac{2B_\omega}{\sqrt{i}kr}\frac{I_1\left(\sqrt{i}kr\right)}{I_0\left(\sqrt{i}ka\right)}e^{i(\omega t-\theta)} \qquad (1)$$

$$\tilde{B}_\theta = i\left(\tilde{B}_r - 2B_\omega\frac{I_0\left(\sqrt{i}kr\right)}{I_0\left(\sqrt{i}ka\right)}e^{i(\omega t-\theta)}\right) \qquad (2)$$

Where $ka = \left(\frac{a\omega_{pe}}{c}\right)\sqrt{(\omega-\omega_e)/\nu_{ei//}}$, a separatrix radius, ω_{pe} plasma frequency, c speed of light, B_ω strength of rotating field, ω frequency of rotating field, ω_e rigid rotation frequency of electron fluid. The variables related to an RMF are taken to be

$$\tilde{A}(r,t) = \overline{A}(r,t)e^{i(\omega t-\theta)}$$

where the higher harmonics are neglected and the time derivative of $\overline{A}(r,t)$ must be is slower than the RMF frequency.

Using the analytical expression of the rotating magnetic field, the time derivatives of the axial magnetic field and the ion azimuthal velocity are respectively written by

$$\frac{\partial B_z}{\partial t} = \frac{\eta_\perp}{\mu_0}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial B_z}{\partial r}\right) - \frac{1}{r}\frac{\partial}{\partial r}(ru_{ir}B_z) - \frac{e}{2m_e}\frac{\overline{\omega}'_e}{\overline{\omega}'^2_e+1}\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\left|\tilde{B}_r\right|^2\right) \qquad (3)$$

$$\frac{\partial u_{i\theta}}{\partial t} = -\nu_{in}u_{i\theta} - \frac{e^2}{2m_e m_i}\frac{\overline{\omega}'_e}{\overline{\omega}'^2_e+1}r\left|\tilde{B}_r\right|^2 \qquad (4)$$

where ν_{in} is the collision frequency with neutral atoms and $\overline{\omega}'_e = (\omega-\omega_e)/\nu_{ei//}$. For simplicity, we neglect the ion radial motion and assume the axial magnetic field is given by

$$B_z(r) = 2B_a\left(\frac{r}{a}\right)^2 - B_a \qquad (5)$$

where $B_a = \frac{\mu_0 en(\omega_e-\omega_i)}{4}a^2$ and ω_i the rigid rotation frequency of ion fluid.

By averaging $\frac{\partial B_z}{\partial t}$ with respect of the radial coordinates, Eqs.(3) and (4) are written by

$$\frac{\partial(\omega_e-\omega_i)}{\partial t} = -\frac{8}{\Gamma}\left(\frac{c}{a\omega_{pe}}\right)^2\nu_{ei\perp}(\omega_e-\omega_i) + \frac{4}{\Gamma}\left(\frac{c}{a\omega_{pe}}\right)^2\frac{\overline{\omega}'_e}{\overline{\omega}'^2_e+1}\frac{e^2}{m_e^2}\left\langle\left|\tilde{B}_r\right|^2\right\rangle_s \qquad (6)$$

$$\frac{\partial\omega_i}{\partial t} = -\nu_{in}\omega_i + \frac{e^2}{2m_e m_i}\frac{\overline{\omega}'_e}{\overline{\omega}'^2_e+1}\left\langle\left|\tilde{B}\right|_r^2\right\rangle_s \qquad (7)$$

respectively.

3. Linear Stability of Equilibrium Solutions

Setting the right hand side of Eqs.(6) and (7) to be zero, we have the non-unique equilibrium solutions as for ω_e and ω_i [4] We consider two limiting cases as follows,

(i) $\left\langle \left|\tilde{B}_r\right|^2 \right\rangle_s \cong B_\omega^2$ for $ka \le 1$ and (ii) $\left\langle \left|\tilde{B}_r\right|^2 \right\rangle_s \cong 6\sqrt{2}\, B_\omega^2 / (ak)^3$ for $ka >> 1$. We obtain the approximate solutions

$$\omega_{e0} = \omega \left\{ 1 - 2\frac{\alpha}{\alpha+1}\left(\frac{\nu_{ei\perp}}{\nu_{ei//}}\right)\left(\frac{\nu_{ei//}}{\omega_{rf}}\right)^2 \right\} \tag{8}$$

for $ka \le 1$, and

$$\omega_{e0} = \omega \left\{ 1 - 18\left(\frac{\alpha+1}{\alpha}\right)^2 \left(\frac{\nu_{ei//}}{\nu_{ei\perp}}\right)^2 \left(\frac{c}{a\omega_{pe}}\right)^6 \left(\frac{\omega_{rf}}{\nu_{ei//}}\right)^4 \left(\frac{\nu_{ei//}}{\omega}\right)^3 \right\} \tag{9}$$

for $ka >> 1$, where $\varpi_{rf} = eB_\omega / m_e$. On the other hand, the equilibrium ion angular frequency is

$$\varpi_{i0} = \frac{1}{\alpha+1}\varpi_{e0} \tag{10}$$

for both cases. Here, $\alpha = \frac{\nu_{in}}{\nu_{ei\perp}}\frac{m_i}{m_e} >> 1$, and $\varpi_{e0} \approx \varpi$ is used in deriving the solutions.

The former gives the solution for the full penetration of an RMF, while the latter corresponds to the partial penetration. The equilibrium of the solution for the partial penetration is more interesting to interpret the present experimental results.

By linearizing Eqs.(6) and (7) around the latter solutions, we obtain the normalized equations for the perturbations $\Delta\varpi_{e(i)} = \Delta\omega_{e(i)} / \nu_{ei//}$

$$\frac{\partial \Delta\varpi_e}{\partial \bar{t}} = -\left[C_3 + (C_1 - C_2)C_4\right]\Delta\varpi_e - \bar{\nu}_{in}\left(\frac{a\omega_{pe}}{c}\right)^2 \Delta\varpi_i \tag{11}$$

$$\frac{\partial \Delta\varpi_i}{\partial \bar{t}} = -\frac{m_e}{m_i}(C_1 - C_2)\Delta\varpi_e - \bar{\nu}_{in}\left(\frac{a\omega_{pe}}{c}\right)^2 \Delta\varpi_i \tag{12}$$

where $\bar{t} = t\nu_{ei//}(c/a\omega_{pe})^2$, $\bar{\nu}_{in} = \nu_{in}/\nu_{ei//}$,

$$C_1 = \frac{3\Gamma}{2}\frac{\nu_{ei\perp}}{\nu_{ei//}}\left(\frac{a\omega_{pe}}{c}\right)^2 \quad C_2 = \frac{1}{36}\left(\frac{\alpha}{1+\alpha}\right)^3\left(\frac{\nu_{ei\perp}}{\nu_{ei//}}\right)^3\left(\frac{a\omega_{pe}}{c}\right)^8 \frac{\varpi^3}{\varpi_{rf}^4}$$

$$C_3 = \frac{8}{\Gamma}\frac{\nu_{ei\perp}}{\nu_{ei//}} \quad C_4 = \frac{8}{\Gamma}\left(\frac{c}{a\omega_{pe}}\right)^2 + \frac{m_e}{m_i}$$

and $\varpi_{rf} = \omega_{rf}/\nu_{ei//}$, $\varpi = \omega/\nu_{ei//}$.

Here, we use the flux conservation relation represented by

$$\left(r_w^{\ 2} - a^2\right)B_a = const. \tag{13}$$

of which effects appear as the variable $\Gamma = \left(r_w^{\ 2} - a^2\right)/\left(2a^2 - r_w^{\ 2}\right)$. We also neglect the perturbations of the density and the temperature, simply because the parameter dependences of their changes in a resistive time scale are unknown. We can derive the eigenvalue equation by setting $\Delta\varpi_e$ and $\Delta\varpi_i$ to be proportional to $\exp\left(\lambda\bar{t}\right)$ in Eqs.(11) and (12). The largest eigenvalue is

$$\lambda = -C_3 - \frac{8}{\Gamma}\left(\frac{c}{a\omega_{pe}}\right)^2 \left(C_1 - C_2\right) \tag{14}$$

The second term which represents the ion spin-up effects is much smaller than the unity and safely neglected. Then, the condition for the stability is $\lambda > 0$, i.e.

$$C_3 > \frac{8}{\Gamma}\left(\frac{c}{a\omega_{pe}}\right)^2 \left(C_2 - C_1\right) \tag{15}$$

Using Eq.(9) and setting $\Gamma = 1$, $\left(1+\alpha\right)/\alpha = 1$ and $\nu_{ei//} = \nu_{ei\perp}$, we have the simple expression

$$\frac{a\omega_{pe}}{c} < 2.1\,\frac{\varpi_{rf}^{\frac{2}{3}}}{\varpi^{\frac{1}{2}}} \tag{16}$$

for the stability. The similar matters are discussed in Ref.[4], which did not include the effects of ion spin-up motions or the radial change of a separatrix. It was concluded that the equilibrium solution representing the partial penetration was always unstable, and could not be realized.

4. Discussions and Conclusions

We quantitatively discuss the results derived in the previous chapter using the concrete FRC and RMF parameters shown in Table 1, which are similar to those of TCS.[2]

Using the largest Eigenvalue given by Eq.(14), we estimate the time changing rate of $\Delta\varpi_e$ and $\Delta\varpi_i$ by $\lambda\bar{t}$ and compare with ωt

$$\frac{\lambda\bar{t}}{\omega t} = \frac{C_3\,\bar{t}}{\omega t} = \frac{8}{\Gamma\varpi}\left(\frac{c}{a\omega_{pe}}\right)^2 << 1$$

Since the dynamical response is much slower than the RMF frequency, then the assumption to take the variables related to an RMF as $\overline{A}\left(r,t\right)e^{i\left(\omega t-\theta\right)}$ is valid.

The substitution of the parameters listed in Table 1 into Eqs.(8) and (9) gives *ka=1.0* and $(\omega-\omega_{e0})/\omega = 0.001$, and *ka=9.5* and $(\omega-\omega_{e0})/\omega = 0.080$, respectively. Since the values $(\omega-\omega_{e0})/\omega$ in both of the equilibrium solutions are much smaller than unity, then the assumption of $\omega_{e0} \cong \omega$ in obtaining the approximate solutions may be justified.

The stability condition given by Eq.(16) is marginally satisfied by the parameters of Table 1. Then, the operation at the partial penetration of the RMF may be stable, at least marginal. It is consistent to the numerical simulation study by Milroy[5].

Since the equilibrium solution which gives the partial penetration of the RMF will become unstable for a denser and larger FRC, of which $a\omega_{pe}/c$ is larger, the full penetration may occur.

The reversal condition of an FRC is $B_a/B_{z0} \approx 1$, where B_{z0} is the externally applied axial magnetic filed strength. Since

$$\frac{B_a}{B_{z0}} = \frac{\omega_{e0}-\omega_{i0}}{\omega} = 0.87 \approx 1,$$

then, the current driven by an RMF is somehow enough to reverse the externally applied magnetic field $B_z=160\ G$.

In conclusion, the paper first studied the RMF penetration into the FRC at the view-point of the stability of the equilibrium between the drag force exerted on the electrons by the ions and the force driven by the RMF. We should study the dynamical behaviors of the penetration in order to clarify the physics of the RMF penetration more directly.

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References

[1] M. Ohnishi et al., Trans. Fusion Technol. **27**,391 (1995).
[2] A.L. Hoffman, Nuclear Fusion **40**,1523 (2001).
[3] I.R.Jones, W.N. Hugrass, J. Plasma Physics **26**, 441(1981).
[4] W.N. Hugrass, Aust. J. Phys. **38**, 157 (1985).
[5]R.D.Milroy, Phys. Plasmas **7**, 4135 (2000).

Table 1 Reference Parameters

$B_\omega = 40 \quad G$	$\nu_{ei//} = 1.1\times10^7 \quad s^{-1}$
$\omega = 0.5\times10^6 \quad r/s$	$\bar{\nu}_{in} = 0.012$
$T_e = 15 \quad eV$	$a\omega_{pe}/c = 150.0$
$n = 1.0\times10^{19} \quad m^{-3}$	$\bar{\omega}_{rf} = 64.5$
$a = 0.25 \quad m$	$\bar{\omega} = 0.045$
$r_w = 0.33 \quad m$	$\alpha = 21.7$
$B_z = 160 \quad G$	$\Gamma = 1.0$
	$\nu_{ei\perp} = \nu_{ei//}$